THE GROUND WORK
OF
INDUCTIVE LOGIC

BY

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REVISED SECOND EDITION

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PREFACE TO THE SECOND EDITION

It is a matter of regret that owing to unavoidable circumstances it had not been possible for me to have the Second Edition of my book on Logic published in due time. I am glad that I am able to make the book available to students now. For a fairly long time the two parts of my book on Logic have been a prescribed text-book for Intermediate students of Calcutta University. In preparing the Second Edition of my book I have modified it in such a way as will make it a suitable text-book of Logic for Intermediate students of all Indian Universities. The object of my book has been to give students a thorough grounding in Logic. I have tried to make this Edition as lucid as possible. All difficult portions have been suitably modified or eliminated. Most of the first chapter of this part has been re-written and the topics have been so arranged as will make it possible for students to follow them without any difficulty whatever. Other chapters have also been suitably modified with the object of meeting the fundamental needs of students. In spite of this the discussion of every topic is adequate and students will find that this edition covers the whole of their syllabus. Exercises with hints have been provided at the end of the book which the students will do well to consult.

I wish to express my thanks and indebtedness to those authors from whose writings I have freely drawn. Whenever I have differed from them I have stated the reason for my doing so. Among these authors I wish particularly to mention the names of Mill, Bain, Joseph, Stebbing, Bosanquet and Welton and Monahan. I must also express my thanks to my much-esteemed teacher Mr. H. H. Crabtree, who is unfortunately no more now. He read through the whole of the manuscript of the First Edition of the book, read through all the proofs and made valuable suggestions for the improvement of the book. As the Second Edition contains much of the improvements made by him I cannot but remember him at the time of having the Second Edition published.

Calcutta, N. N. SEN GUPTA.
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CHAPTER I

DEFINITION, NATURE AND SCOPE OF INDUCTION

Introductory Remarks

In the first part of my book on Logic I have indicated, in a general way, the relation between Formal or Deductive and Material or Inductive Logic and also between formal and material truth. It has been pointed out that there is an intimate connection between Deductive and Inductive Logic and they represent two aspects of thought. At the outset of our discussion of Induction, it is necessary to understand clearly the relation between Deduction and Induction. We should therefore do well to open our discussion on Induction with a classical example to show how it is related to Deduction. In Induction we argue thus:—Socrates is mortal, Plato is mortal, Aristotle is mortal, and other men died, therefore, all men are mortal. In Deduction (the perfect form of which is Syllogistic reasoning) we argue thus:—All men are mortal, Socrates is a man, therefore, Socrates is mortal. If we compare these two forms of inference, we find that in Induction we pass from particular facts of experience to a universal real proposition. Thus the conclusion of Induction is more general than the premises. But in Deduction the conclusion cannot be more general than the premises. Usually, in Deductive inference, the conclusion is less general than the premises, though sometimes it may be as general as the premises. The two exam-

Relation between Induction and Deduction explained in a general way by concrete examples

ples of Induction and Deduction given above show that the conclusion of the Induction in this particular case is the major premise of the syllogism. This shows that there is an intimate connection between Deduction and Induction. But as Deduction is concerned merely with formal truth, Deductive reasoning can be carried on with the help of symbols, as in the case,—M is P, S is M, ∴ S is P. On the other hand as Induction is concerned with material truth, its premises and conclusion must have reference to actual facts. Though Induction is concerned with material truth, it does not ignore formal truth. The conclusion established by Induction and its premises must be materially true and the process of reasoning must be formally true. With these preliminary remarks we may now pass on to the consideration of the relation between Induction and Deduction in detail.

Induction and Deduction

Nature of syllogistic reasoning

We have remarked that iction a *connected* processes of thought. We must now try to find the proper relation between them and to define the nature of induction proper. We have found that the syllogism is the most important type of formal reasoning. Syllogistic reasoning is hypothetical in character, that is, in it the conclusion is true if the premises are true. When we argue from the premises, "All material bodies gravitate", and "All stones are material bodies" to the conclusion that "All stones gravitate", the conclusion is true if the premises are true. The same remark applies to the argument "All men are mortal, All kings are men, therefore, All kings are mortal." But deduction is not concerned with the actual truth of the premises. Thus the conclusion of a syllogistic argument may be formally valid,

since it may follow logically from the premises, without being materially true. But the aim of logic is to establish not simply formal *validity* but *truth* as well. Let us take the syllogism, "All birds lay eggs, All bats are birds, ∴ All bats lay eggs". Here the reasoning is formally valid, but the conclusion is not true. Why is the conclusion false? It is false because the minor premise, "All bats are birds", is materially false. But *deductive* reasoning cannot establish the material validity of its *premises* which are accepted as true, whether they are really so or not. What then can we do to establish the truth of the premises and of the conclusion of a piece of deductive reasoning? Is it possible for us to establish the truth of syllogistic reasoning by means of prosyllogisms? Supposing we argue that "All men are mortal", "Socrates is a man", ∴ "Socrates is mortal", because "All animals are mortal", "All men are animals", ∴ "All men are mortal", and supposing we prove this latter syllogism by the syllogism, "All living beings are mortal, All animals are living beings, ∴ All animals are mortal", and so on. Is it possible for us to prove the truth of the first syllogism, "All men are mortal, Socrates is a man, ∴ Socrates is mortal", in this way? Obviously such a process involves an *infinite regress* and cannot establish the truth of the original syllogism. What method is there, then, of assuring ourselves of the truth of syllogistic reasoning? We have found that at least one of the premises of every syllogism is a universal proposition or principle, and that the truth of the conclusion depends mainly upon the truth of this universal proposition or principle; that is, if the *conclusion* of a *syllogistic* argument is to be both *valid* and *true*, the process of *reasoning* must be *valid* and the *premises* must be *true*. A syllogism can legiti-

The truth of the Syllogism cannot be established by prosyllogisms

mately be true and thus achieve the end of reasoning, only if the principle or universal proposition from which it starts can be proved true, if the minor premise also is true, and if the process of reasoning is correct.

How can we establish the truth of the universal proposition which is a premise of the Syllogism

The solution of our problem depends on the use of *induction* to establish the *validity* of the universal *premise* which is involved in a deductive argument. If the major premise of a syllogism is a self-evident axiom, its truth is intuitively perceived, and then it requires no proof. Thus in the argument, "Two contradictory propositions cannot both be true (major premise), 'S is P' and 'S is not P' are contradictory propositions (minor premise), ∴ 'S is P' and 'S is not P' cannot both be true (conclusion)," we can be certain of the truth of the conclusion, because the major premise from which the argument starts is the principle of contradiction, which is universal, necessary and self-evident, and because the reasoning is valid. But the premises with which most syllogistic reasonings are carried on are not self-evident, and their truth has to be established by induction. Let us take the syllogism, "All empires decay, Britain is an empire, ∴ Britain will decay." This argument will be valid and true if the major premise 'All empires decay' is true and the reasoning is sound. How can we establish the truth of the *major premise*? We can do so if by *observing* different instances of empires which have decayed, we can discover a *necessary connection* between the properties of empire and the attribute of decaying. Thus the universal premise of a syllogism, provided it is not an axiom, can be proved to be true by

The universal premise of a syllogism, unless self-evident, has to be proved by induction

induction, that is, by a generalisation from experience. In induction we establish a universal proposition by observing particular facts of experience and by discovering a universal and necessary connection between the subject of inference and the inferred property.

Induction as an Inverse Process

The discussion of the question whether induction is an *inverse process* will throw further light upon the problem of the relation between deduction and induction. Jevons, Bosanquet, Ward, Whewell, Welton and others regard induction as an *inverse process*. By this they mean that while deduction is a natural process, induction is an unnatural process of reasoning, since the process involved in induction is the opposite of the process of reasoning involved in deduction. In other words, the inductive process is the converse of the deductive process. Aristotle distinguishes between the *order of nature* and the *order of experience*; in the order of nature, the general principle is prior to the sensible fact; in the order of experience, the reverse holds good. In induction we *pass from particular facts to general principles*. Thus after observing the mortality of A, B, C, D, E and others, who are men, we pass on, by induction, to the mortality of all men. But nature begins with principles and produces particulars afterwards. In deduction we follow the natural order, and pass on, say, from the mortality of all men to the mortality of A, B, C, D and others, since they are men. In other words, nature passes from the whole to the parts, and this order is followed in deductive reasoning; whereas induction passes from the parts to the whole and is thus an inverse or unnatural process. It may further be pointed out that while deduction passes from the reason to the

Inverse process explained

consequence, induction passes from the consequence to the reason. Therefore it is argued that whereas in deduction we pass from the antecedent to the consequent or from the premises to the conclusion, in induction we pass from the consequent to the antecedent, or from the conclusion to the premises. For all these reasons induction is supposed to be unnatural or an inverse process. Because the universal, though it manifests itself in particulars, is logically prior to them, therefore the process of reasoning which passes from the particulars to the universal must, it is said, be regarded as an unnatural or an inverse process, for it treats particulars or facts of experience as though they were prior to the universal.

(Jevons points out that deduction should be regarded as prior to induction because the conclusion of induction which is a mere supposition or hypothesis can be proved true only by deductive verification. So Jevons holds that induction is dependent on deduction.)

(Mill, on the other hand, in discussing the value and validity of syllogistic reasoning, points out that the syllogism is hypothetical in nature. Its conclusion is true if its premises are true, but the syllogism does not prove the truth of its premises. The truth of the major premise of the syllogism, if it is a universal, real proposition, can be proved true only by induction. So, according to Mill, induction is prior to deduction and the function of deduction is to apply the universal conclusion established by induction to particular cases.)

But the above views confront us with the question whether form is prior to matter, or the universal to particulars, or the whole to its parts or whether particulars are prior to the universal, or matter to form. Reality con-

tains both form and matter, both the universal and the particulars, and it is impossible to determine which is prior and which is posterior. The whole exists in its parts, form in matter, and the universal in particulars; and it is useless to discuss which is prior and which is posterior in logic. Men reason deductively from form to matter, or from the universal to the particulars, as well as inductively from matter to form, or from particulars to the universal. *Both* deductive and inductive processes are natural from the *logical* point of view, and the question whether form is prior to matter or matter to form should be left to the metaphysician for decision.

From the logical point of view both deduction and induction are natural processes

Forms of Induction Proper or Imperfect Induction

Logicians have recognised mainly four forms of induction which are: (1) Scientific Induction, (2) Induction by Simple Enumeration, (3) Analogy, (4) Argument from Probability. We may now consider these forms of Induction Proper. The above forms of Induction Proper are sometimes called *imperfect inductions* as opposed to *perfect induction*.

1. Scientific Induction or Induction Defined:

Induction is regarded as the *process of thought by* means of which *laws* are *discovered* and *established*. It is therefore "the operation of discovering and proving general propositions". Mill defines it as "that operation of the mind by which we infer that what we know to be true in a particular case or

Mill's view of induction explained

cases, will be true in all cases which resemble the former in certain assignable respects.'' Thus, according to Mill, to establish a general proposition by means of which we can state a law, we must *observe* some *instance or instances*. After such observations we can pass from what we have observed to a *universal* proposition, which includes both *observed* and *unobserved* cases on the ground of *similarity*. Mill, therefore, holds that when a general proposition is arrived at by observing some particular instances, we pass from here and now to the not-here and the not-now, and that induction thus always involves a certain 'hazard or leap'. Further, he holds that what makes this passage from the known to the unknown possible is likeness or similarity between instances observed and instances unobserved, which are comprehended under the general proposition established by induction.

In induction we pass from observed particulars to a general proposition or from some observed particular to an unobserved particular on the ground of similarity

Further, according to Mill, in induction we can pass, on the ground of similarity, not only from observed particulars to a general proposition, but also from an observed to an unobserved particular. Thus by means of induction we may either establish the general proposition that boiling water can destroy animal life, after we have observed a number of instances of boiling water destroying animal life, or we may argue that since boiling water has destroyed animal life in one case, it will destroy it in another particular case as well.

Mill's view of induction is, on the whole, correct. But we may also define induction as a *process of reasoning in*

which by observing or experimenting upon particular facts of experience, we discover a uniform and necessary connection of cause and effect, between the subject of inference and the inferred property on the basis of which we can establish a universal and real proposition.

Our definition of scientific induction

The conclusion of an induction is therefore a universal, necessary, and real proposition. Such a proposition must be true always and in all circumstances, and this is possible only when it expresses a necessary relation of content between the subject and the predicate of the proposition. Such a universal and necessary proposition can be established if all the *conditions* of scientific induction are fulfilled. Thus from the observed mortality of some men we can pass on to the mortality of all men, if we can ascertain the conditions upon which human mortality depends. Thus a scientific generalisation can be expressed by a universal proposition, either in the form 'S is P', or in the form 'If S is M, it is P'. Further, the universal propositions established by induction must be *real* or *synthetic* and not verbal or analytic propositions. If the general proposition established by induction is merely a summary of observed instances, it is not an induction proper, since then the general proposition does not go beyond the observed particulars, and thus does not contribute to the advancement of knowledge. Further, in order to establish a universal, necessary and real proposition, we must observe and experiment upon facts very carefully, if experiment is possible, in order that we may discover the conditions which determine their occurrence. Scientific observation and experiment therefore require the *analysis* of facts by means of which we can separate the *essential* from the non-essential conditions.

Explanation of the expression universal, necessary and real proposition

The aim of this analysis is to find out a *causal connection* between the subject of inference and the inferred property. Only when we can, by means of analysis, discover the conditions under which the given phenomena occur, that is, only when we can find out their *cause*, can we establish a scientific induction. (The nature of causation and of observation and experiment will be discussed in the next two chapters.)

Observation and experiment enable us by the analysis of facts to establish causal connection upon which scientific induction rests.

We may illustrate all this by means of a concrete example. Suppose that we want to establish the proposition that land is subject to the operation of the law of diminishing returns, (which means that if the number of units of capital and labour applied to a certain piece of land is gradually increased, the land tends to yield a less than proportionately increased return). We must observe how different pieces of land behave when more and more capital and labour are employed upon them. We may also experiment by employing capital and labour on a parcel of land ourselves. Thus by carefully observing the behaviour of land in various circumstances, we may establish a causal connection between the employment of capital and labour on land and its productive capacity, which will enable us to establish the law of diminishing returns.

There may be a legitimate induction from the observation of one instance only

Mill says that the observation even of *a single instance* may suffice to establish an inductive generalisation; only one observation may suffice to reveal a *causal connection* between the subject of inference and the inferred property. Induction is possible

because form exists in matter, and the universal in particulars, but it requires very great caution and very careful analysis of facts to discern this form in matter or this universal in particulars.

Observation of a large number of instances is not indispensable to induction

It is often supposed that in order to establish a universal proposition we must observe a large number of instances, but this is not indispensable. If by observing only one instance or a few instances we can discover the required law, there is no need to multiply them. A large number of instances are useful only when they enable us to discover the causal law in a better way.

Conceptions or definitions not to be regarded as inductions

We have already stated that an induction is a universal, necessary and real proposition arrived at on the basis of observation and experiment. Such *conceptions* as justice, humanity, goodness, virtue, etc., should not be regarded as *inductions* despite the fact that they cannot be formed in advance of observation, because conceptions are not propositions. Nor should a *definition* be regarded as an induction, for when we define a term we state its connotation and such a proposition is *analytic* or verbal, not synthetic or real, whereas an *induction* is a real or synthetic proposition.

2. Induction by Simple Enumeration:

The process of reasoning in which a large number of instances are observed or enumerated as the necessary basis of an inductive generalisation is called *induction by simple enumeration*. It is a popular form of induction as distin-

guished from scientific induction. According to induction by simple enumeration, induction rests upon uncontradicted experience. Two very famous examples of such inductions are, 'All crows are black', and, 'All swans are white'. Such generalisations are made after observing black crows only and white swans only in a large number of cases, and observing no instance to the contrary. But such generalisations have been falsified by the discovery of crows and swans of other colours. Thus, the *counting of instances should not be regarded as the essence of induction*, whose *formal ground* is really the *principle of causation.*

Simple enumeration cannot establish the scientific proposition

We shall discuss the nature of induction by simple enumeration more fully in another chapter. It is the common-sense or popular conception of induction, as opposed to scientific induction. Simple enumeration is connected with the theory of probability or calculation of chances.

3. Analogy:

Analogy is also a popular form of induction as distinguished from scientific induction. The nature of analogy will be fully discussed in a subsequent chapter. Here we may briefly indicate the distinction between scientific induction and analogy. According to Mill, in induction we may pass from particular instances to a universal, real proposition or from one particular instance to another particular instance, on the ground of similarity. Thus if we argue from the mortality of some men to the mortality of all men or from the mortality of Ram to the mortality of Hari, it is a case of induction. In analogy also we pass from the particular to the particular on the ground of similarity. Thus, if

we argue that because Ram who is tall and fair is intelligent, Shyam who also is tall and fair must also be intelligent, our argument is analogical. But when we pass from the mortality of Ram to the mortality of Hari, we can be sure of the truth of our conclusion. But when we pass from the intelligence of Ram to the intelligence of Shyam as in the above example, we cannot be sure of the truth of our conclusion. What then are the main points of difference between induction and analogy? In induction, the points of similarity upon which it rests are important; but, in analogy, the points of similarity on which it rests are superficial. Secondly, we discover causal connection between the subject of inference and the inferred property in induction, but no such causal connection is discovered in analogy. So while the conclusion of induction is certain, the conclusion of analogy is doubtful.

4. Argument from Probability:

The theory of Probability is also an inductive doctrine. Its nature will be clearly explained in a subsequent chapter. Probability rests upon calculation of chances. Conclusions arrived at by probability cannot be as certain as those arrived at by induction. Statistical sciences are based upon calculation of chances. The conclusions of probable arguments are true on the average and in the long run. Thus, when a statistician makes a forecast about the possible amount of rainfall in a particular year or the possible yield of crops in a particular year, his conclusion cannot be regarded as absolutely accurate. Similarly, when a life insurance company calculates the longevity of men of a particular nationality living in a particular geographical area, the conclusion which it arrives at is true on the average and is not absolutely accurate. The theory of Probability is an inductive doctrine because the conclusion of a probable

argument rests upon observation of facts, and cannot be as certain as that of induction.

Processes of Reasoning improperly called Inductive or Processes Simulating Induction

There are certain processes of reasoning which are sometimes regarded as inductive, though they are not really so. The essence of induction consists in the passage from the known to the unknown, but in these processes there is no such passage. There are mainly three types of such processes of reasoning, *viz.*, (1) Perfect Induction, (2) Colligation of Facts, and (3) Mathematical Reasoning. We may now consider these processes of inference one by one. They are sometimes called "processes simulating induction".

1. Perfect Induction:

Perfect or summary induction defined

Perfect induction is that form of reasoning in which after observing *every member* of a *group* we pass on to the conclusion that what is true of *each* of them is true of the *group as a whole.* The form of such reasoning is—X, Y, Z, etc., are B; X, Y, Z, etc., are all A; ∴ All A is B. Suppose, after observing that every planet shines by the light of the sun, we come to the conclusion that all planets shine by the light of the sun, our argument is, to use the scholastic terminology, perfect induction. Again, if after observing that the cow, the sheep, the deer, etc., which together form all the species of horned animals, ruminate, we conclude that all horned animals ruminate, our argument is a complete, summary or perfect induction. Similarly, if after observing John, Peter, Matthew and every other Apostle of Jesus Christ,

we conclude that all Apostles are Jews, then the argument is a perfect induction. Perfect induction is called induction by complete enumeration, because in it a generalisation is made after observing or enumerating every instance which comes under it. It is to be remarked that this 'perfect' induction is not *induction* in the proper sense of that term. Here there is no passage from the known to the unknown and what we do is merely to sum up certain observed instances. If it is to be called *induction* at all, it would be better to describe it as summary induction, as Johnson does.

A perfect induction cannot establish a universal real proposition; it involves no inductive leap. Perfect induction can never establish such a law as 'All material bodies gravitate'. The conclusion of a perfect induction is not wider than the instances observed.

Induction and Colligation of Facts

Whewell's view of induction as colligation of facts

Whewell holds that induction is colligation of facts. He maintains that in induction, by observing a number of instances, we *colligate* or *unite* them under a suitable notion. Thus, after observing a number of instances we form a hypothesis to unite them under it. Such colligation is, in his view, the essence of induction, since in it we have to exert our mental faculty to superimpose the conception upon the facts.

Mill does not regard colligation as induction proper

Mill however thinks that colligation is not induction proper, and that Whewell confuses *description*, which is colligation of facts, with induction. Thus, Mill says

that, when after circumnavigating a piece of land we say that it is an island, we colligate our observed experiences under a suitable notion, and that this is mere description, not induction. When we go round a place in a boat and discover that the place is surrounded on all sides by water and is therefore an island, we no doubt colligate observed facts, but such colligation is not induction. Similarly, he says that Kepler's discovery of the elliptical orbit of the planet Mars is no doubt colligation, but it is not induction. It was known before Kepler that this and other planets return to the same place after moving for a period. Kepler, having observed the different points through which the planet Mars moved, colligated his experiences under the notion that the movement of Mars is elliptical. This is mere description, and not induction. Induction, Mill says, requires colligation of facts, but colligation is not necessarily induction. *Induction*, according to him, requires *description*, *explanation* and *prediction;* in colligation we only describe observed phenomena, we neither explain them nor can we make any prediction from them. Induction must *pass* from the *known* to the *unknown*, but mere colligation cannot accomplish this passage from the observed to the unobserved.

Colligation is mere description and cannot secure the passage from the known to the unknown

Nature of mathematical reasoning.

The controversy between Whewell and Mill is due to the fact that whereas Whewell regards induction as the science of discovery, according to Mill it is primarily concerned with proof and only secondarily with discovery. Mill's view seems, on the whole, to be more satisfactory, since colligation only enables us to form hypotheses, but induction is concerned as much with proof as with

the formation of hypotheses or discovery. Induction must not only bring the observed instances under a suitable notion, but it should also enable us to pass from instances which have been observed to instances which have not yet been observed.

Induction and Mathematical Reasoning

Nature of mathematical reasonings

Mathematics has for a long time been regarded as deductive in character, but some have held that it is based upon generalisation from experience. In geometry we are concerned with spatial relations. Suppose we argue that what is true of equilateral, isosceles and scalene triangles is true of all triangles, or that since a circle, an ellipse, a hyperbola, and a parabola cut a straight line at two points, every conic section will cut a straight line at two points; should such arguments be regarded as inductive? Strictly speaking they are similar to what is known as *perfect induction* or *induction by complete enumeration*, which is *deductive* in nature. In such reasoning there is no inductive 'leap', no passage from the known to the unknown. Since equilateral, isosceles and scalene triangles exhaust the possible forms of triangle, what is true of all of them will necessarily be true of all triangles. Similarly the circle, ellipse, hyperbola and parabola exhaust the possible forms of conic sections, and therefore what is true of them must necessarily be true of all conic sections. Again, when we argue from the fact that the three angles of an observed triangle are equal to two right angles, that the sum of the angles of every triangle is equal to two right angles, our argument, though it appears to be inductive, is really

Some mathematical reasonings analogous to perfect induction

deductive in character. Such an argument, says Mill, is *improperly called induction by parity of reasoning*. We really assume that all triangles have fundamentally the same nature with regard to certain qualities. This is contained in the definition of a triangle, and therefore when we say that what is true of a particulár triangle regarding the sum of its angles is true of all triangles, our argument really proceeds from this fundamental assumption, and is properly speaking demonstrative or deductive. In such a case, says Joseph, "the reasoning proceeds directly from the apprehension of certain necessary relations among characters in the subject of our study to the apprehension of other relations seen to be bound up with those; not, as in induction, from the observation of facts to belief in the only connections with which they cannot be shown to be incompatible." This remark holds good of the reasoning involved in arithmetic and algebra.

Some mathematical arguments are improperly called induction by parity of reasoning

Complete and Incomplete Induction

The distinction between complete and incomplete induction is confusing and hence unimportant in logic. We have found, in course of our previous discussion, that scholastic logicians have regarded perfect induction as complete induction as it rests upon complete enumeration of facts which are summed up in the conclusion. But other logicians have regarded scientific induction as complete induction and they have used the term incomplete induction to indicate such processes of inductive reasoning the conclusions of which are uncertain. So by incomplete induction they understand such processes of reasoning as induction by simple enumeration, analogy and probability. There seems to be another view according to which every form of induction proper whether

scientific or unscientific should be regarded as complete induction. Thus we find that the connotation of the terms complete and incomplete induction is not clearly fixed.

Aim of Induction; Scientific Method

We may now consider after having discussed the nature of induction, how it is related to our knowledge of the world.

Reality and ordered universe

It is our belief that *reality* or the *universe* is an *order;* that within this world-order various *systems* co-exist; and that the mind can discover *laws* which can be arranged in an orderly hierarchy, connecting facts and giving them meaning and revealing that the universe is rational. The universe is not merely rational and ordered; but it is at the same time capable of being known by the human mind.

Laws are the forms which exist in material objects. Without matter they are empty. Aristotle long ago recog-

Form and Matter and Laws

nised the fundamental truth that forms exist in matter, or universals manifest themselves in particulars, and that one without the other is a meaningless abstraction.

If the things of the world were absolutely separate and unconnected, scientific knowledge would have been out of the question. We have knowledge of the world, however incomplete or imperfect that knowledge may be, because the world exhibits order. The impulse of the scientist would have died out if it were impossible to believe that the things of the world are governed by laws. When we classify objects into species and genera, we are guided by this belief that within the ordered universe various systems exist, and that everything in the world can be referred to one or other of these systems.

Every science employs Deduction or Induction or both. Induction a scientific method par excellence

Though the sciences are not branches of logic, yet it is undeniable that every *science* follows a *method*, and induction has not improperly been regarded as the scientific method *par excellence*. Most sciences rely on a combination of the deductive and inductive methods, though some are pre-eminently deductive and others pre-eminently inductive. Mathematics and sciences like pure physics in which mathematical calculation plays a large part, are pre-eminently *deductive* sciences. The greater the progress a science achieves, the more does it become deductive in principle. But the natural sciences, which start with *particular facts* and *after observing* and *experimenting* upon them, attempt to discover and establish the general principles which underlie them, are properly regarded as *inductive* sciences. Such sciences are pre-eminently amenable to inductive procedure. Since all sciences except the mathematical ones are empirical in character, depending upon observation and experiment to discover and establish truths, induction in common parlance is called the scientific method. The inductive sciences, however, cannot dispense with certain formal principles, nor can deductive sciences avoid material considerations. The social sciences in particular require the help of both deduction and induction. These sciences sometimes, argue from first principles to particular facts, and sometimes start from particular facts to discover principles afterwards.

Is Induction a science of discovery or of proof ?

There was a long-drawn controversy between Whewell and Mill on the question whether *induction* is a method of *discovery* or of *proof*. Whewell held that induction is mainly a method of discovery because it aims at discovering laws after examining particular facts of experience. According to Mill, on the other hand, it is mainly a method of proof, being primarily con-

cerned not with the discovery of laws but with their establishment by experimental methods. According to Whewell the main object of induction is the formation of legitimate *hypotheses*, whereas according to Mill it is the *establishment* or *proof* of hypotheses.

It is concerned with both

We may, however, observe that though it is an important function of science to discover laws, it is equally concerned with proving them, since it cannot rest satisfied with mere conjectures. Accordingly, induction as the scientific method *par excellence* is equally interested in both discovery and proof.

Scientific Thinking

Scientific knowledge is ordered knowledge

"Scientific thinking is controlled and directed thinking; it is essentially methodical." We know that scientific thinking is systematic. An orderly arrangement is what is usually meant by a system. A system contains ordered elements. "A set of elements exhibits order when, given the properties of some of the members of the set, the properties of the other members of the set, or at least of some of them, are thereby determined." Thus 1, 2, 3, 4; 1, 2, 4, 8, are orders. "Thus an order is a relation that orders the members of a class, *i.e.*, a set of elements, in a certain way." An army is an ordered set of men. There may be various kinds of order. A routine of work is an order. Life without order is chaotic. Scientific thinking essentially consists in the organisation or co-ordination of the facts with which it deals. The sciences are concerned not with facts as such, but with ordered facts. In science there is required a certain kind of attitude to the facts and a certain kind of predominantly logical method. We have two characteristics that belong to science: the selection

The meaning of order

of a certain kind of facts and the use of a certain kind of method. It is the applicability of the logical method to the facts of the external world that makes the natural sciences scientific. Mathematical sciences are deductive, as we have already pointed out. Empirical or natural sciences are inductive. A proposition asserted by a natural science can be tested by comparison with observable facts. A mathematical proposition may sometimes be independent of what happens to exist. But both mathematical and natural sciences provide order. The data of the natural sciences are observable facts, and a proposition which expresses a fact of the external world can be proved by observation.*

Some General Remarks

We know that logic is mainly concerned with thought and logical thought always involves conscious effort. Idle reverie or dreaming in which there is no conscious effort is not thought in the logical sense. Logical thinking is reflective thinking, not passive recognition. Reflective thinking is always purposive, and aims at solving a problem. Mere association of ideas is not inference, since in it no conscious effort is necessary. In judgment we perceive reality directly, while in inference the reference to reality is indirect or mediate. The ground of inference is some concrete universal, not an abstract universal, since in inference we have always to discover identity in difference or the whole in parts. In inference we are always conscious of the operation of the universal, and therefore, simple recognition, as when we recognise a man whom we saw in the past, is not inference. Even in inductive inference the univer-

The concrete universal is always the ground of inference, which requires the perception of identity in difference

*For this section students are referred to Miss Stebbing's *Modern Introduction to Logic*, ch. 13.

sal consciously operates as when we pass from a number of observed particulars to a universal proposition, or from the particular to the particular. In inductive inference, there is a temporal transition in passing from the known to the unknown. We can clearly recognise in it the operation of the concrete universal. In all inference we must perceive identity in difference or homogeneity in diversity. Reality is concrete and it manifests itself in diversity, and logical thought consists in acquiring the knowledge of such a reality. Mere discovery is an accident of reasoning, its essence is proof. Inference always involves abstraction and selection, because without such mental activities we cannot perceive identity in difference, and without perceiving such identity we cannot infer. Mathematical logic reduces inference to implication, and aims at simplicity and homogeneity instead of complexity and diversity. But both these elements are necessary in inference. Inference also involves comparison and discrimination, without which selection is not possible. Selection often necessitates omission, but there may be logical selection without involving omission to make inference possible. In every inference, whether deductive or inductive, there is conscious operation of the concrete universal.

Inference involves selection, comparison and discrimination

Though inference requires the help of perception, yet it should be distinguished from the latter, even if perception may require the aid of past experience. Inference should also be distinguished from recollection and psychological suggestion, because in them, as in perception, conscious effort in the logical sense is not present. "The possibility of inference depends both upon the logical relations holding between the propositions and upon the relation of

The nature of inference further considered

the thinker to these propositions"; that is, inference has two aspects—epistemic or subjective and constitutive or objective.

Two conditions of inference

But implication may exist between propositions without being apprehended. Inference requires the assertion of a proposition by a thinker. 'If P then Q', asserts neither P nor Q; whereas 'P, therefore Q' asserts both P and Q.

Inference requires a relation of implication and assertion

Inference requires both a relation of implication between the propositions constituting a piece of inference, and an assertion of this relation. In other words, in inference the premises must imply the conclusion, and the conclusion as well as the premises should be asserted. Miss Stebbing says that "the possibility of inference is conditioned by what the thinker knows and what is true as well as by the logical relations between propositions and not between a proposition and a thinker". The statement of implication is if P then Q, since Q is implied by P; while the statement of inference is, P therefore Q, since in such a case there is assertion in addition. The relation of implication is like the relation which exists between Ground and Consequent.*

Note:—Deductive, Inductive and Social Sciences:—Mathematics and Pure Physics are deductive sciences. The following sciences are regarded as inductive, *e.g.*, physics, chemistry, botany, biology, zoology, psychology, etc. Mechanics, astronomy, being mathematical sciences are deductive. So are geometry, algebra, and arithmetic deductive sciences. Economics, politics, history, sociology, etc., are social sciences.

*Students are referred to Bosanquet, Johnson, and Stebbing for the above section.

CHAPTER II

POSTULATES OF INDUCTION.

Uniformity of Nature.

For a very long time it has been held that the principles of Uniformity of Nature and Causation are the postulates, presuppositions, or formal grounds of induction. It is held that without the assumption of these principles inductive generalisation is impossible, that is, that these assumptions are the ground of induction. The real foundation of induction, however, is *causal uniformity*, and therefore not every form of uniformity can be regarded as its formal ground. We have discussed (in Part I, Introduction, Chapter 3) those fundamental principles of thought which are the universal postulates of logic, *viz.*, the law of identity, the law of contradiction and the law of excluded middle. We have now to consider the postulates which are specially necessary for induction.

The principle of causal uniformity is the ground of induction

We have already seen that the scientist believes that reality or the universe is an ordered system in which the different occurrences are connected by laws. It is also the faith of the scientist that the order of Nature, that is, the laws in which Nature manifests herself, is intelligible. If he did not believe this he would not venture to search for the laws of Nature. So Miss Stebbing says that in Nature "what happens happens in accordance with laws, and these

laws are such that we can discover them". Some philosophers hold that science also assumes that the world is a unity and that in it there is identity,

Identity, Persistence and Continuity in the universe

persistence and continuity. The universe throughout is one, but manifests itself in diversity; in it there is identity amidst difference, persistence amidst change; and the universe is a continuous whole, the parts of which are intimately related with one another. It is also supposed that the

The principle of simplicity

world is simple, that is, its complexities are reducible to simple principles. This demand is aesthetic, and as the world is simple it is not only knowable but is also beautiful. Since, according to

The universe provides us with regulative principles of thought

this belief, the world is not chaotic but is a system and is intelligible, it provides regulative principles of thought which make methodology possible.

But in science we can do without the wider conception of unity and can get on with the less general conception of the *uniformity of Nature.* This principle is stated in various ways, *e.g.*, Nature is uniform in her operation, Nature repeats herself, the future will resemble the past, the absent is like the present, the universe is governed by laws, etc. Its essence is that *what happens under certain conditions will always happen under the same conditions.* The conditions under which fire warms or water

The world not only uniform but also multiform

quenches thirst will always be the same. The principle of the uniformity of Nature manifests itself in various uniformities or laws, so that the world is said to be not only uniform but also multiform. The follow-

ing laws, for example, may be regarded as generically different: Material bodies gravitate, water quenches thirst, fire warms, men pursue pleasure, etc. If Nature is really uniform in her operation, it may be asked, why do we find what looks like caprice and accident? Is not the behaviour of weather and climate fitful? Can we predict human actions? The world abounds in irregularities as well as in regularities. To this objection we may reply that what appears to be accidental can be brought under wider and more general laws. This means that what is irregular in relation to one law is found to be regular in relation to some wider law. Welton and Monahan instance the following interesting example:—"Lord Rayleigh and Sir William Ramsay found that two specimens of nitrogen, each obtained by a different method, had slightly different atomic weights. Relative to the available knowledge, the atomic weight of nitrogen was a capricious quantity, but relative to the knowledge which their experiment enabled them to obtain, the capriciousness disappeared, for the differences in weight were found to be due to the presence of argon, a gas previously unknown'.

Nature is uniform in spite of the appearance of irregularity

The question that will be asked is all-important with regard to induction, *viz.*, How does it come about that *sometimes a single instance justifies us in making an inductive generalisation, whereas at other times a large number of instances may fail to establish a valid induction?* To answer this question is to solve the problem of induction. Mill rightly pointed out that it is possible because what happens under certain conditions always happens under the same conditions. So if these *conditions of occurrence* can be dis-

Mill's problem

covered, we can establish a valid induction from one instance only. But Mill's theory of induction is open to the same charge of *petitio principii* as he levelled against the syllogism. He says that the ground of induction, *viz.*, the uniformity of Nature, is itself an induction. After observing in many instances that under certain conditions certain events occur, we expect that the same events will always occur under the same conditions. Thus he says that the principle of uniformity of Nature is itself a generalisation from experience, and is not even one of the first generalisations which men make, but rather one of the last. But if the ground of induction is itself an induction from experience, how can we establish the validity of induction on the basis of this principle? Mill begs the question. The validity of the uniformity of Nature has first to be established before we can establish the validity of the inductive generalisations which are based upon it. This is what is known as the *Paradox of Induction*.

Paradox of Induction.

Intuitionists on the other hand hold that the principle of the uniformity of Nature is an intuition. This means (1) that the belief in Nature's uniformity exists from the beginning in the human mind, that is, it is given ready-made but that it is manifested by experience. Though the knowledge of this principle exists in the mind, we become clearly aware of it only when we experience uniformities in the world. It means, (2) that without the postulate of the uniformity of Nature no scientific knowledge is possible, and therefore it is the presupposition of all sciences. The logical justification of the principle of uniformity of Nature is that

The Intuitionist view.

The Principle of uniformity of Nature is the pre-supposition of all scientific knowledge

it provides guidance to scientific enquiry. It is a *regulative principle* of thought, and the presupposition of all scientific knowledge.

Three types of Uniformity

Three types of *uniformity* are recognised, viz., (1) that of *succession*, the main form of which is the principle of causation; (2) that of *co-existence;* (3) that of *equality*. 'Fire burns' and 'water quenches thirst' are examples of causal uniformity, and properly speaking the principle of causal uniformity or the principle of causation is the ground of induction. Examples of uniformity of co-existence are the fusibility, lustre, opacity, etc., which are co-existing properties of metal, and the inertia and gravity which are co-existing properties of all material bodies. 'Things equal to the same thing are equal to one another' is an example of uniformity of equality. The mathematical sciences, which are based upon the relations between numbers, rest upon this uniformity.

Different forms of co-existence

Co-existence admits of different forms. Mr. Carveth Read enumerates the following:—(1) *Geometrical* co-existence, which is illustrated in such cases as, If the opposite angles of a four-sided figure are equal, then the opposite sides are equal and parallel. (2) *Universal* co-existence of *properties in substances*, as in the case of the co-existence of inertia and gravity in all material bodies. (3) Co-existence due to *causation*, *e.g.*, when two things co-exist in the same room because they have been placed there. (4) Co-existence of *properties* in a *species* due to its nature, *e.g.*, animality and rationality co-exist in all men, or mind and body co-exist in all animals. (5) *Unique* co-existence, *e.g.*, co-existence of whiteness with milk. Bain rightly

points out that the proof of the uniformity of co-existence is uncontradicted agreement, which is also a condition of the proof of the causal uniformity. Any uniformity which is not causal must stand on its own independent evidence. The coincidence of gravity with inertia has been proved over the entire globe.

The Principle of Causation

The statement of the principle of Universal Causation

There is one law, says Mill, which is co-extensive with the entire field of successive phenomena; *viz.*, the law that *every fact which has a beginning has a cause.* 'The notion of cause being the root of the whole theory of Induction, it is indispensable that this idea should, at the very outset of our enquiry, be, with the utmost practicable degree of precision, fixed and determined'. Empirical sciences are not concerned with the ultimate cause of the world, but with causal laws which are the relations between events in time and space. The principle of *causation* is *universal* because every event is causally connected with some other event. The causal relation, according to Mill, is a temporal relation, that is, the cause is prior to the effect in time. According to Bain, the principle of universal causation may be stated thus: *Every event that happens is definitely and uniformly connected with some prior event or events, which happening, it happens, and which failing, it fails.* This law is often expressed by the following two clauses: (a) *every event* has a *cause;* (b) the *same effect* always follows from the *same cause.*

We often speak of the laws of causes in the plural. When we speak of causal laws we mean that in the universe there is a plurality of causal uniformities, and these laws are

the species of the principle of universal causation. We have already noted that the principle of causation is a species of the principle of the uniformity of Nature.

Aristotle's view of causation

**[Aristotle distinguished four kinds of causes, *viz.*, the *material* cause, the *formal* cause, the *efficient* cause and the *final* cause. We may explain his view by an example. Let us take a statue: its production requires marble, which according to Aristotle is the material cause; a design or idea in the mind of the sculptor, which is the formal cause; the power acting to produce the statue, *viz.*, the manual energy and skill of the sculptor and other forces which he employs—this power is the efficient cause; and the end or motive which prompts him to act, which is the final cause.]

We shall now give an account of Mill's view of causation and consider how far he has succeeded in providing a view of causation on the basis of which induction can establish universal and necessary propositions from experience.

Mill regarded cause as an *invariable* and *unconditional antecedent.* He agrees with Hume that a cause in relation to its effect is an antecedent which produces the effect in time. The causal relation therefore, according to both Hume and Mill, is a *temporal* relation, the *cause* being *prior* to the *effect* in time. But not every prior antecedent can be regarded as the cause. Suppose an earthquake in India is followed by various economic, political and other changes in different parts of the world. We cannot regard the earthquake as the cause of these changes. So Mill, like Hume, says that the cause must be an *invariable antecedent* to the effect, that is, the *same cause* must *always* be followed by the

same effect. If fire warms to-day and cools to-morrow, then fire cannot be regarded as the cause of warmth. Again if a particular poison is the cause of a man's death, then whenever that poison is injected into the human body, it must be followed by death.

The meaning of condition

But according to Mill the cause must not merely be an invariable antecedent. It must also be *unconditional*, that is, it must be adequate to explain the effect. Here Mill goes beyond Hume. A condition is that which plays some part in bringing the effect into being. According to Mill a condition may be either *positive* or *negative*. A positive condition is the presence of a fact without which the effect cannot happen. A negative condition is the absence of a fact which frustrates the occurrence of the effect. Cause, therefore, according to Mill, is the sum total of conditions, positive and negative, which are necessary to bring about an effect. Thus if we know all the conditions which are required to explain a given effect, we can establish a necessary relation between cause and effect. This has long been regarded as the scientific view of causation. It may be illustrated by an example. Suppose some one is attacked by fever and dies. In this case it will not be sufficient to say that fever is the cause of death, for many men who are attacked in the same way do not die. What then is the cause of death? The presence of fever, the man's bodily condition at the time, etc., are the positive conditions of his death; but the negative conditions have also to be considered, such as the absence of necessary medical aid, good nursing, etc., which might have prevented the effect under question. So cause is the totality of conditions

The cause is the sum total of conditions, positive and negative

positive and negative. A condition, therefore, is a part of a cause; it is all the conditions together that constitute the cause. So Mill says that if a man takes mercury and exposes himself and consequently catches cold, it is not sufficient to say either that the taking of mercury or that exposure is the cause of catching cold. To explain the phenomenon in question, both these conditions, and any others which might have contributed to bring the effect into being, have to be mentioned. The growth of a plant depends upon various conditions, such as the properties of the seed, the conditions of the soil, heat, light, climate, etc., and the absence of counteracting conditions.

The meaning of agent, patient and collocation

Mill points out that the distinction between agent and patient in a causal occurrence is unsound. The earth is supposed to be an agent when it attracts a stone, and the stone a patient which is attracted by the earth. But Mill rightly points out that the stone may also be regarded as an agent, for the stone attracts the earth as the earth attracts the stone. When fire melts a piece of wax the fire is regarded as the agent and the wax as the patient, but in the process of melting the fire is no more active than the wax: it could not melt wax unless the latter were responsive to action. It cannot melt a piece of stone, because the stone is not similarly responsive. A cause or an effect may consist of a certain collocation. When an article of food is taken, the result may be a certain collocation in the body which may be the cause of a disease. When a builder builds a house, the result is a certain collocation or arrangement of bricks, sand, mortar, wood, etc.

Plurality of Causes

The doctrine of plurality of causes explained

Can there be a *plurality of causes?* Can the *same effect* be produced by *different causes?* The practical man thinks that it can. Thus it is said that death may be produced by a gunshot wound, by drowning, by hanging, by different diseases and so on. Similarly light, it is said, may be produced by different agencies, such as the sun, the moon, electricity, gas, a lighted candle, and so on. Thus from the standpoint of pragmatic logicians or common men, the causal relation is a many-one and not a one-one relation. But there are others according to whom the conception of cause requires that the same effect should always be due to the same cause, that is, the *causal relation* is *reciprocal* and *one-one*, not many-one.

Criticism of the doctrine

Why then does the common man think that different causes may produce the same effect on different occasions, that is, that plurality of causes is a legitimate notion? It is neglects to analyse the total causal situation and total effect situation. If we *specialise the effect*, we discover that the lights produced by different agencies, such as the sun, the moon, electricity, etc., are not really the same. We can also avoid this difficulty of plurality by *generalising the cause*. If different lights have essentially the same characteristics, then the causal characteristics which produce them, though they appear to be different, must be essentially the same. Thus the causal property which produces the same effect must necessarily always be the same. Welton and Monahan say: 'If each effect of a certain kind were considered in all its particularity, we should see that one particular effect is not pro-

duced by a plurality of causes, but by one only'. 'We may say then, that if we refer to general properties, there may be plurality of causes, but if we refer to concrete cases of particular groups of properties, it is improbable that there ever is plurality of causes'. If so, then the causal relation is reciprocal or one-one, and not many-one.

Quantitative Mark of Causation

Causation viewed from the standpoint of conservation of energy

Some logicians have tried to introduce a *quantitative* relation between cause and effect by associating the conception of *conservation* of *energy* with the conception of *causation*. Physicists assume that the total amount of energy or force is constant; that is, there cannot be any absolute destruction of energy, though one form of energy may reappear in another form. So Bain, who views causation from the standpoint of conservation of energy, says, "The great generalisation of recent times, variously designated the conservation, persistence, correlation, convertibility, equivalence, or indestructibility of energy, is the highest expression of cause and effect." The statement of the law is that "Force, energy, moving power is embodied in various forms, all mutually convertible at a definite (fixed) rate. The extinction of energy in one form is accompanied by the creation of energy in another form; in the transmutation work is said to be done, and no force is absolutely lost."

Potential and kinetic energy

A distinction is drawn between *kinetic* energy and *potential* energy. Matter in *motion* is matter with force manifested as actual, apparent, or *kinetic* energy, while matter in *position* is matter with force

manifested as *potential* energy. Gunpowder manifests itself as kinetic energy when fire is applied to it and causes an explosion, but it is an example of potential energy when it is at rest, that is, when it does not manifest itself in an explosion. Energy therefore may be either active or inactive. Though the total amount of energy is constant, mechanical force may pass into heat, and heat into mechanical force, and so on. Thus we find that heat produced in various ways imparts movement to machinery, and in this way one form of energy reappears in another form without sustaining any loss. The sum of the kinetic and potential energy within this material system is constant and unalterable. This means that force or energy is persistent. Whatever the striking body loses, the struck body gains. "If two inelastic bodies encounter and arrest one another's movements, the mechanical or molar energy is indeed sunk; but reappears in an equivalent energy communicated to the molecules, and manifested as heat". Mechanical force may give rise either to mechanical force or to molecular force such as heat, chemical force, electricity. Potential energy may be transformed into kinetic energy, or vice versa. Thus when a stone is placed on a table it manifests itself as force in position, but if the support is withdrawn and the stone moves towards the ground it manifests itself as kinetic energy, that is, energy in motion.

From the standpoint of conservation of energy the effect is regarded as equivalent to the cause

"Causation, viewed as conservation, is thus the transferring or re-embodying of a definite amount of force." When we say that heat causes mechanical motion, or that electric motion causes light, we really mean that one form of energy reappears in another form. In such a case there is quanti-

tative equivalence between cause and effect, that is, the quantity of energy lost by the causal phenomenon is gained by the effect phenomenon. Potential energy is also called energy of situation, arrangement or collocation. Gunpowder is a concentration of potential chemical energies or of combinable elements in a situation of readiness to combine. "There is a vast amount of potential motion in the universe in the form of gravitation." Similarly coal underneath the earth contains a large amount of potential energy.

Potential energy and collocation

Mill rightly observes that strictly speaking the notion of conservation of energy does not introduce any change in the scientific notion of causation. The scientific view of causation which we have set forth before may be regarded as the sufficient ground of induction, and there is no need of associating the conception of conservation of energy with that of causation. Mill says that whether the phenomenon is called a transformation of force or not, it has its own set or sets of antecedents, with which it is connected by invariable and unconditional sequence. In some cases of causation, however, the principle of conservation of energy may clarify the relation between cause and effect. But a quantitative relation between cause and effect cannot be established in all cases of causation. When we say that an insult causes pain or virtuous actions cause happiness, it is not possible for us to establish quantitative relations between cause and effect.

Quantitative relation between cause and effect cannot always be established

The universe is complex, and it seldom happens that one effect is produced by one cause only. Several causes

Composition of causes explained

usually co-operate to produce a complex effect. Where the complex effect is a mechanical phenomenon, it is nothing but the sum of the separate effects produced by different causes. Such a case of causation Mill calls *composition of causes*. He writes: "I shall give the name of the composition of causes to the principle which is exemplified in all cases in which the joint effect of several causes is identical with the sum of their separate effects." Thus the joint weight of two things is the sum of their separate weights. Similarly the joint motion produced by two forces acting together is equal to the sum of the separate motions produced by these forces acting separately. These are examples of *homogeneous* intermixture of effects, the complex effect being of the same nature as the separate effects. In such cases of causation, computation or calculation is possible. The laws of such complex effects can be deduced from the laws of their separate effects.

Heteropathic intermixture of effects

Another form of joint effect is exemplified in chemical and physiological phenomena. When the combination of two particles of hydrogen with one particle of oxygen gives rise to water, the *result* is *different* in nature from the causes which combine to produce it. Similarly the phenomenon of life, which is a complex effect, cannot be explained by analysing the parts of the animal body.

Such joint effects are called *heteropathic* intermixture of effects, and the law of the joint effect cannot be deduced from the laws of the effects of the separate causes.

Note 1: Marks of Causation:—

Cause, we have seen, is according to Mill, an invariable and unconditional antecedent to the effect. So the qualitative marks of cause are the attribute of being an antecedent, the attribute of being invariable and the attribute of being unconditional. Some logicians say that immediacy is also a mark of cause as it is an immediate antecedent to the effect. We have, however, seen that immediacy cannot be regarded as a mark of cause. Apart from qualitative marks a quantitative mark of causation is recognised when it is viewed from the standpoint of the conservation of energy, *viz.*, cause is equal to the effect in quantity.

Note 2: Mutuality or Reciprocity of Cause and Effect:—

Sometimes the relation between cause and effect is mutual or reciprocal, *i.e.*, what is cause may be regarded as the effect and what is effect may be regarded as the cause. Thus moral degradation may be the cause of poverty and poverty may be the cause of moral degradation. Again thrift may encourage industry and also industry may encourage thrift. Progress in civilisation may be the cause of educational progress and educational progress may be the cause of progress in civilisation. Such examples illustrate what is meant by mutuality or reciprocity of cause and effect.

CHAPTER III

OBSERVATION, EXPERIMENT

(Material Grounds of Induction)

An induction, as we have seen, is a universal, necessary and real proposition, arrived at from experience of particular facts in conformity with the principle of causation, which is regarded as the formal ground of induction. In order to establish general propositions or laws, scientists have to start with *observation* of and *experiment* upon particular facts. Since observation and experiment provide materials for induction, they are sometimes regarded as the *material grounds* of induction.

Induction starts with observation and experiment

No induction is possible apart from the discovery of causal uniformities connecting facts together. But the phenomena of the world are very complex, and it is often a difficult task to eliminate irrelevant circumstances in the effort to discover causal connections. So phenomena have to be observed with great care and caution. A man cannot be a good scientist if he lacks the power of observation. *Observation* requires the power of *analysis*. In order to observe accurately a complex phenomenon, we have to break it up into simpler elements, to eliminate those circumstances which have no bearing on the question

Nature of observation

Observation requires analysis

at issue. This power of analysis is possessed in very different degrees by different men. Mill rightly observes that an observer is not one who merely sees the thing which is before his eyes, but he who sees what parts that thing is composed of.

To be a successful observer, a man must be *free from bias or prepossessions.* Bacon rightly pointed out that bias or prejudice is a great enemy of accurate and scientific observation, and warned the investigator against the four kinds of prepossessions or 'idols'. Different kinds of uncriticised beliefs very often stand in the way of right observation. "Before the discovery of oxygen, bodies when heated were thought to give off a special substance named 'phlogiston'. When it was found that metals so heated became heavier, the apparent contradiction was resolved by supposing phlogiston to have a negative weight which diminished the true weight of compounds in which it was present". This example shows how bias retards the progress of science. Owing to the acceptance of this false phlogiston theory, the progress of chemistry was checked for a long time.

The elimination of prejudice or bias is necessary

Men commit fallacies not only from mal-observation but also owing to non-observation. We often fail to observe facts which it is necessary to observe if we are to arrive at a correct view. This is often due to carelessness and also to prepossessions. The bacteriological theory of disease could not have been formulated before bacteria were observed with the help of powerful microscopes. The cause of malaria was supposed to be the marshiness of the areas in which it was prevalent, but more careful observation removed this wrong notion, and proved that marshy places are mala-

Non-observation and mal-observation

rial because mosquitoes are abundant in them, the poison of which is the real cause of malaria. (See the chapter on Fallacies).

Right observation is the result of previous knowledge of a subject

A man who has previous knowledge of a subject can observe a fact *more correctly* and accurately than a person who has no such knowledge. Newton could not have discovered the law of gravitation if he had had no previous acquaintance with physics. The fall of an apple meant much more to him than to an ordinary man. A man acquainted with a subject can better observe and estimate the significance of a phenomenon relevant to the subject than a lay man can. A machine, writes Welton, means much more to an engineer than to the man in the street. For successful observation depends upon special aptitude.

Correct observation also requires insight and selection

Understanding of a fact depends not only upon previous knowledge but also upon the *insight* of the observer. Great inventors have been geniuses who have known what to observe and how to observe and could discriminate between what is relevant and what is irrelevant to a particular matter. We have noted that observation requires analysis, but mere analysis is not sufficient for scientific purposes. A scientist has to *select* circumstances which are important and eliminate those which are unimportant. Without selection causal laws cannot be discovered. By means of selection points of similarity in two situations are discriminated from the points of difference. Selection is very often aided by hypothesis, which is sometimes formed by analogy. Observation which involves analysis and selection is often guided by purpose.

Observation aided by instruments

For accurate observation men use *instruments* invented by scientists, such as the balance, thermometer, microscope, telescope, etc. Only scientists who have knowledge of these instruments can often discover new facts. The microscope has acquainted us with many minute objects which were previously unknown. Accurate analysis of objects becomes possible with the aid of these instruments. The delicate instruments invented by Sir Jagadish Chandra Bose have made minute and accurate study of the activities of plants possible. "We can judge weight by the hand, but not with the accuracy of the balance. We can distinguish temperature by the skin as greater or less, but not with the certainty of the thermometer." Scientific instruments have greatly aided the progress of knowledge. "If simple observation is largely dependent upon previous knowledge, still more is this the case with observation made with the aid of scientific instruments, for all such instruments embody in themselves much knowledge." A good observer should be acquainted not merely with one particular branch of science but with different branches of it. To make a perfect observer an extensive acquaintance is requisite, not only with the particular science to which his observations relate, but with every branch of knowledge which may enable him to appreciate and neutralize the effect of extraneous disturbing causes.

Correct observation depends on correct inference

From all that we have said it is clear that correct *observation* rests upon correct *interpretation* of facts and upon right *inference*. Observation involves inference, and without valid inference scientific observation is not possible.

Observation by experiment

Observation by *experiment* has greatly aided the progress of science. Experimental sciences are supposed to be the most exact. "The use of scientific instruments is one aspect of the transition from simple observation to experiment." *Observation* passes into *experiment* only when the *facts* to be observed can be *modified at will*. In experiment facts are under our control, while in observation we depend upon the presentation of facts by Nature. Thus observation, even though it is performed with the aid of instruments, cannot be regarded as experiment, if it introduces no change in the phenomena observed. Jevons gives the name of Natural Experiment to that kind of observation in which we observe a thing from different positions or from different angles. But such observation should not be called experiment, for in this process the facts observed are not modified. The distinction between observation and experiment, however, is only one of degree and not of kind, because experiment is nothing but observation of facts under conditions produced by the observer at will. Observation passes into experiment when we can manipulate the facts as we like. So Bain says that observation is finding a fact, while experiment is making one.

Observation and experiment differ only in degree

Experimental sciences are progressive

Physics and chemistry are experimental sciences, and they have made much progress because physical and chemical phenomena can be subjected to experiment. But a biologist can do comparatively little in the way of experiment to establish his theories; a historian cannot experiment upon past events; a sociologist, an economist or a political thinker cannot make

Not all sciences experimental

use of experiment to any great extent; an astronomer also must rely upon observation to discover the law of the behaviour of celestial bodies.

Some characteristics of experiment

Experimental sciences are not only progressive but also *accurate*. In experiment we can carry the facts to be observed home with us as it were, and can observe them with coolness under varying circumstances. Experiment therefore enables us to eliminate irrelevant circumstances accurately and with great success, making the discovery of causal laws easier. In experiment we can even create new facts. We can by means of experiment observe the falling of a body in a vacuum. Similarly the chemist in his laboratory creates many compounds which nature does not afford. The facts of the world are so complex that they cannot be rightly interpreted and understood without the aid of experiment. Experiment, wherever it is possible, is always preferable to observation. It can establish *quantitative* precision. The aim of both experiment and observation is the same, *viz.*, the discovery of the conditions under which phenomena occur. Experiment,

Experiment enables us to establish reciprocal propositions

by eliminating irrelevant circumstances and finding out the conditions under which events occur, enables us to establish such reciprocal propositions as, If S is M, it is P, and If S is P, it is M, which science requires.

We are now in a position to sum up the advantages of observation and experiment over each other. In many sciences men have to depend entirely or mainly upon observation. Thus we cannot have recourse

to experiment in investigating the laws of planetary motion; the movements of the planets cannot be modified and manipulated according to our convenience. A geologist has to depend upon observation to find out the conditions of the formation of rocks. A biologist also, as we have already seen, can comparatively seldom take the help of experiment. Doctors and politicians have very limited scope for experiment. A doctor is not allowed to study the effect of a deadly poison on the human organism by injecting it into the body of a man. A politician can hardly study the effect of war by undertaking a war. Thus while *observation* is possible in *every* science, the field of *experiment* is *limited*.

Advantages of observation over experiment

In observation men can study the cause by observing the effect, and they can also study the effect after observing the cause. In experiment they can try the effect of a given cause, they cannot however try the cause of a given effect; that is, by experiment they can produce the effect, but they cannot produce the cause with the help of the effect. But it is also true that by means of experiment we can discover the cause by studying the effect. Thus a doctor can, by a post-mortem examination, find out the cause of a man's death.

Advantages of experiment over observation

When experiment is possible, it has many advantages over observation. By experiment we can produce a phenomenon which we want to study carefully as many times as we like. Complex phenomena can be minutely studied by means of experiment, which thus enables us to find out the exact relation existing between facts. In experiment we can observe at

leisure and with close attention the effect of a certain cause, by eliminating irrelevant circumstances. Experiment provides the means by which we can adapt the facts to the case in hand by producing the sort of variation that we need. Another advantage of experiment over observation is that it enables us to produce a phenomenon in known circumstances and surroundings, so as to take account of all extraneous influences. In observation we have to study what Nature may present to us. We cannot observe natural phenomena as we like. If we wish to study the nature of a comet, we may have to wait for months or years for its appearance. Moreover, natural phenomena are very often fleeting, and the passage of some occurrences is so swift that we can hardly observe them. The sciences would not have made so much progress if they had had to depend solely on observation in studying the facts of the world. Our knowledge of electricity would not have made such progress if experiment had been impossible. Thus experiment provides exact knowledge and contributes a great deal to the advancement of science. Experimental sciences are more progressive than those which depend for their materials upon observation alone.

Inductive Procedure

Induction begins with observation of and experiment upon facts

We may conclude this chapter with a brief account of the way in which induction proceeds to establish valid conclusions. We have seen that the materials of science are provided by *observation* and *experiment*. So induction starts with observation of, and experiment upon, facts, and the examination of testimony in cases in which we depend for our knowledge upon the experience of the past. We have sufficiently discussed the nature of these

methods in the preceding sections. We have seen that observation, experiment and examination of testimony yield successful results only when we can *analyse* the given facts correctly and minutely. The *result* obtained by these processes should be stated clearly and accurately, that is, it should be well *defined* by *accurate* statements.

After going through these processes scientists often form Hypotheses

We may sometimes pass direct from observation and experiment to scientific laws, but very often after studying facts we have first to form hypotheses, which are suppositions regarding the conditions under which the facts in question occur. But these suppositions about causal laws have to be *verified* or proved. Mill thinks that hypotheses can be *proved*, and causal connection established accurately, by the help of the inductive *methods* or *canons* which he sets forth. According to him these canons are the instruments by means of which irrelevant circumstances can be eliminated and causal laws precisely established. But we shall find later that Mill's canons are really deductive in nature, as they are deduced from the principles of elimination. *Deduction* therefore is the main method by means of which men verify the hypotheses they form. When hypotheses are verified, inductive logic is able to attain its object, that is, it can establish the universal, necessary and real propositions by means of which the laws of Nature are stated.

Mill's canons and deductive method

Our procedure

In the next chapter we shall consider the nature of hypothesis, and in the chapter following we shall discuss how hypotheses are formed. In connection with the problem of the verification

or establishment of hypothesis we shall then consider Mill's inductive canons and deductive method. We shall then consider the nature of probability, and proceed to the consideration of explanation, which is the goal of induction, since induction aims at explaining facts of experience. In the fifth Book we shall be engaged with the discussion of the problem of Method, its general nature and the methods employed by different sciences.

CHAPTER IV

HYPOTHESIS

Nature and Definition of Hypothesis

Factors which influence scientists in interpreting the facts of the world

"Scientific method is the means by which we seek to answer questions about the order of Nature" (Stebbing). The scientist seeks to explain the facts of Nature by discovering the links which bind them together. We have seen that observation, experiment and testimony inform us of the facts with which the scientist starts in his investigation, and these facts have to be interpreted if he is to find their true place in a system of knowledge. Further every investigator is influenced by the 'climate of opinion', which consists of the total outlook of educated men. Everyone has in addition some 'local weather' or 'personal equation', that is, individual opinions or habits. All these factors determine how different men will explain the phenomena of Nature.

The preliminary stage of explanation is by means of hypothesis

Popular and scientific hypothesis

Hypothesis is a stepping stone to discovery and induction

Before laws can be established, it is often necessary to make suppositions or guesses which may explain the facts. Hence it is said that the initial form of an explanation is a supposition, and such a supposition as to the relation between given facts is called a *hypothesis*. The hypotheses formed by ordinary men are often wild guesses, but those of the scientist are the result of careful observation and experiment. Thus a man well acquainted with a particular department of knowledge or with different sciences is likely to form a hypothesis which may ultimately prove true, while the guesses of uneducated and ordinary men often prove futile. Hypotheses therefore when rightly formed are a stepping stone to the discovery of inductive generalisations.

Even scientists often have to reject hypothesis after hypothesis before they arrive at the right one. Kepler had to reject nineteen hypotheses before he found the right one regarding planetary motion.

Mill's definition of hypothesis

Mill defines hypothesis as "any supposition which we make (either without actual evidence or on evidence avowedly insufficient) in order to endeavour to deduce from it conclusions in accordance with facts which are known to be real, under the idea that if the conclusions to which the hypothesis leads are known truths, the hypothesis itself either must be or at least is likely to be true." But this definition

of hypothesis is not perhaps accurate. A scientist does not make a hypothesis without evidence or on insufficient evidence. He forms it after careful observation and experiment. An ordinary man may try to explain a natural occurrence as the work of a spirit or some other supernatural agency, but a scientist never does so. The scientist always weighs as much evidence as he can find before forming a hypothesis. Mill however is right in saying that the object of forming a hypothesis is to establish it by deducing conclusions from it and comparing those conclusions with facts. If these deduced conclusions agree with facts, then the hypothesis is taken to be true; if they do not so agree, it is taken to be false. A hypothesis therefore has to be proved or established by deduction and observation. Thus the proof of a hypothesis involves both the deductive and the inductive method.

A criticism

Mill minimised the importance of hypothesis, supposing that from observation and experiment men could directly pass on to inductive generalisations or laws with the help of the experimental methods or canons which he enunciated. But he recognised that men often pass to inductive generalisations after framing hypotheses from observed facts. Mill's attitude towards hypothesis was due to his regarding induction as a science of proof and not of discovery. Whewell on the other hand supposed that the main function of induction was to form legitimate hypotheses which could be proved by deduction. Whewell held this view because according to him discovery is the main function of induction, which he regarded as 'colligation of facts'. This work of colligation should not be regarded as

Mill's attitude towards hypothesis

Whewell's attitude towards hypothesis

something distinct in its nature from the framing of hypotheses. Joseph says that induction is not merely inference, but the whole process by which generalisations are made. Whewell is right in so far as he holds that the colligation of facts, or framing of hypotheses from observed facts, is an important part of induction; and Mill is right in so far as he holds that the framing of hypotheses is not induction. A scientist has to frame legitimate hypotheses and to prove them by appealing to experience. So discovery is as much a part of induction as proof. We have already remarked that a person who is acquainted with different branches of knowledge can form good hypotheses, but this requires imaginative insight as well. Only geniuses such as Galileo, Newton, Einstein have been able to frame hypotheses which have led to great discoveries.

The true nature of induction

Some confusion of ideas arises from the failure to distinguish certain terms such as fact, hypothesis, law and theory. The term 'fact', as we have already stated, is usually used to indicate objects or events, that is, objects of experience. But a law of lower generality is also called a fact in relation to a law of higher generality. A hypothesis, we have seen, is a supposition made to explain facts. When a hypothesis is proved by deduction and observation, it is called a law or a theory. Thus a proved hypothesis is a theory or a scientific law. So we speak of the theory of gravitation, the undulatory theory of light, etc. But the term 'theory' is also used to indicate a system of laws, *e.g.*, the theory of physics, the theory of astronomy, the theory of chemistry etc.; and in unscientific language an unproved hypothesis is also called a theory, *e.g.*, the phlogiston theory in chemistry, the Ptolemaic theory in astronomy, etc.

The terms fact, hypothesis, law and theory explained

Some Examples to Explain the Nature of Hypothesis

Pasteur formed a hypothesis about the causes of disease among silkworms after thoroughly studying all that was known on the subject. He then proved his hypothesis by microscopic examination. His success was due both to his thorough knowledge of the subject and to careful observation. A hypothesis may originate in accidental fashion. "The laws of the internal structure of crystals were suggested to Houg by his observing that in a crystal accidentally broken the fracture showed regular geometrical faces." This hypothesis also was afterwards verified and proved by further observation. Again when it was found that Uranus deviated from its calculated orbit, a hypothesis was formed that this deviation was due to the influence of some unknown planet. Afterwards this hypothesis was established, when the planet Neptune, which attracted Uranus and caused deviation in its movement, was discovered by a powerful telescope.

Wrong hypotheses generally precede right ones, frequently in the same mind. Hypotheses direct our investigation and lead us to the discovery of laws. A scientific hypothesis never ignores facts but always tries to explain them. If it fails to explain the phenomena of Nature for which it was formed, it can never rise to the status of a theory. A hypothesis framed to account for the cause of malaria attributed it to the marshiness of certain places, but further investigation showed that some places which were free from marshes were also malarial. This led to the discovery of the real cause of malaria, *viz.*, the poison of a particular type of mosquito. Thus we find that a wrong hypothesis may direct investigation to the discovery of a causal law. Similarly the Copernican or heliocentric theory in astronomy started from a criticism of the Ptolemaic or geocentric theory, which ex-

plained the movements of the heavenly bodies by cycles and epicycles. The corpuscular hypothesis of light guided investigation and led to the establishment of the undulatory theory of light. We thus find that a hypothesis that is ultimately rejected need not be useless for the purpose of scientific discovery.

We thus find that the formation of a good hypothesis requires knowledge and insight, but even wrong hypotheses can lead to right ones.

Kinds or Forms of Hypothesis

It has long been customary to distinguish three forms of hypothesis, *viz.*, (1) hypothesis about a *law*, (2) hypothesis about an *agent*, (3) hypothesis about a *collocation*. The last two are classed together by some logicians as hypothesis about *cause*, and contrasted with the hypothesis about law. Miss Stebbing brings all these three forms under what she calls *explanatory* hypothesis. Let us first explain them separately:—(1) When we explain the motion of the heavenly bodies by the law of gravitation or the phenomena of light by the corpuscular or by the undulatory theory, our hypothesis is said to be about a *law*. (2) When irregularities in the orbit of Uranus are explained by reference to Neptune, or the phenomenon of light by reference to ether, or the fall of bodies by reference to the earth, our hypothesis is said to be regarding an *agent*. (3) When the position of a particular planet or the phenomenon of an eclipse is explained by reference to relative motions of the heavenly bodies, which is an arrangement of different elements, the hypothesis is said to be about a *collocation*.

Hypothesis about a law

Hypothesis about an agent

Hypothesis about a collocation

Hypotheses may also, from a different point of view, be classified as (a) *working, descriptive* or *provisional* hypotheses and (b) *analogical* hypotheses. (a) A hypothesis is regarded as working or provisional before it has been proved. Investigators often have recourse to such hypotheses for the guidance of their enquiry. Provisional hypotheses often lead to the discovery of true ones. The Ptolemaic hypothesis was a descriptive hypothesis which provided a geometrical representation of the movements of the heavenly bodies. "The hypothesis of the ether as a frictionless fluid and as a completely elastic solid must be regarded as descriptive." The electric theory of matter is also a descriptive hypothesis. Such hypotheses are only provisional. Working hypotheses are frequently assumed provisionally in order to furnish a description of facts, when knowledge is not sufficiently advanced to admit of a true explanatory hypothesis. Huxley recommends the formation of working, descriptive or provisional hypotheses, since any hypothesis is better than none, and may lead to the discovery of very useful laws. Several descriptive hypotheses may function at the same time, some of which may afterwards be found sufficient to explain the facts.

Working, descriptive, or provisional hypothesis

(b) The last kind of hypothesis is *analogical* hypothesis. Miss Stebbing writes that a descriptive hypothesis may develop into an analogical hypothesis. In such a hypothesis we suppose that what is true of one set of phenomena may be true of another, owing to the fact that the two sets have certain formal properties in common. It is by analogy that the formula of the inverse square which applies to gravitation is extended to electrical attraction. Between these two sets of phenomena, *viz.*, gravitation and electrical attraction, there is a structural identity.

Analogical hypothesis

Such hypotheses are psychologically valuable in the process of discovery. Hypotheses condition the process of experimental enquiry, which may lead to induction proper.

Conditions of Valid or Legitimate Hypothesis

A valid hypothesis should explain facts

Hypotheses must fulfil certain conditions, if they are to satisfy the logician. Of these conditions that which is of supreme importance is: (1) A *valid* hypothesis should adequately explain facts by reducing them to order. A hypothesis is formed in order to establish connection between facts, and if it fails to do so, it cannot be entertained in science. A hypothesis therefore should be adequate to explain all the facts which it embraces. The more comprehensive a hypothesis is, the more likely is it to be true. The theory of gravitation is such a comprehensive hypothesis: it explains certain relations not only between terrestrial phenomena but between heavenly bodies also. This first condition, the implications of which we have considered, was recognised by Jevons, who also recognised the following three conditions of legitimate hypothesis, which follow from the first:—

A hypothesis should be self-consistent and in harmony with established laws of Nature

(2) A hypothesis should be *self-consistent* and *in harmony with established laws of Nature*. But a hypothesis need not always be consistent with what are regarded as established laws of Nature. The Copernican hypothesis in astronomy was inconsistent with the then accepted Ptolemaic hypothesis, yet the former has been accepted as legitimate and the latter is now regarded as inadequate to explain the planetary system. Similarly

the wave theory of light, though it conflicted with the corpuscular theory which was once supposed to be the true one, has been adopted as necessary for the explanation of certain optical phenomena, though others appear to be explicable only on the corpuscular theory. Though Einstein's theory of relativity has modified the long-standing Newtonian theory of gravitation, it is supposed by scientists to be legitimate. It is only by the acceptance of new hypotheses that science progresses to new discoveries. The more perfect the sciences become, the less need will there be to form hypotheses which conflict with established laws of Nature.

A hypothesis should furnish a basis for rigorous deductive inference of consequences

(3) A hypothesis should furnish a *basis* for rigorous *deductive inference* of consequences. We cannot deduce anything from what is unknown, or from a mere abstraction. This may be expressed by saying that a hypothesis should be of a *vera causa* or a real cause. "Our hypothesis must be always in accordance with some analogy, or based on some experience; otherwise we can draw no conclusions from it." This condition forbids us to explain natural phenomena by supernatural agencies or by metaphysical entities. We cannot entertain as a hypothesis such an abstract conception as that of a perfectly frictionless liquid, because it is not analogous to anything we know and is not based on experience. Nothing can be deduced from such a hypothesis, and it does not admit of deductive verification. But the hypothesis of ether is about a *vera causa* or real cause, because the conceptions of elasticity and motion can be applied to it by analogy from what we know of the physical universe and also because it renders certain observed facts explicable. Further this condition requires that a hypothesis should be clear and definite. A hypo-

thesis which admits of mathematical expression will be accurate, definite and clear, *e.g.*, Newton's law of gravitation. A vague or indefinite notion cannot be accepted as a hypothesis by scientists.

The inferred consequences should be in agreement with fact

(4) The *consequences inferred* from a hypothesis should be in *agreement with fact*. This condition also is necessary to establish the validity of a hypothesis. A hypothesis states a relation existing between phenomena of Nature. So if it is to be accepted, it must be in agreement with the system of Nature. To prove a hypothesis, therefore, consequences should be deduced from it rigorously and not loosely, and comparison between these consequences and facts should be made with great caution. "Nevertheless, a hypothesis is not to be hastily abandoned on the first *prima facie* conflict with reality."

The above conditions are indispensable to the legitimacy of a hypothesis. But there are certain other conditions, fulfilled by certain hypotheses, which may also be regarded as conditions of legitimate hypothesis. We may consider them now.

A good hypothesis can usually be extended to new regions

(5) A good hypothesis not only explains the facts which it proposes to explain, but it may also explain facts which were originally beyond its range. Thus a good hypothesis can be extended to new regions and will afford a basis for *prediction*. This is what Whewell calls *consilience of induction*. Thus with the help of the principle of gravitation prediction was made of the existence of

Consilience of Induction

Neptune. The same principle served to explain the relation between planets, though it was originally proposed as an explanation of certain relations existing between terrestrial phenomena. Whewell has shown how the wave theory has been extended to new regions. But though prevision or prediction is no doubt a condition of a legitimate hypothesis, it is not an absolute condition.

A legitimate hypothesis should be able to establish its claim over rival hypotheses

(6) A legitimate hypothesis should be able to establish its claim over rival hypotheses. There may be two or more rival hypotheses in the field to explain a set of phenomena, and one of these may ultimately be accepted as valid. Thus the Copernican hypothesis has been accepted in preference to the Ptolemaic hypothesis. *A crucial instance* is an instance which can be explained by only one among several contending hypotheses. It is called crucial because it establishes the claim of one among such hypotheses.

Crucial instance

An experiment by which the claim of one hypothesis among others is established is called a crucial experiment (experimentum crucis).

Crucial experiment

"The term is adopted from Bacon, who tells us that it is transferred from the crosses (or finger-posts) which are put up at crossways to mark and point out different ways." The claim of the undulatory theory of light was apparently established against the corpuscular theory of Newton by a crucial experiment. If the undulatory theory be true, light must move more slowly in a dense refracting medium than in a rarer one; but the Newtonian theory assumed that the attraction of the dense me-

dium caused the particles of light to move more rapidly than in the rare medium. Experiment showed that light moved more slowly through a glass than through air, contrary to the Newtonian hypothesis. Foucault in 1850 "by determining the velocity in both air and water showed that the velocity of light is inversely proportional to the refractive index of the medium." "According to the wave theory light should have a greater velocity in vacuo than in material media; according to the corpuscular theory the contrary should be the case." But the above experiment established the claim of the wave theory in preference to the corpuscular theory. It should be noted however that modern physicists have discovered that there are certain phenomena which they are only able to explain on the corpuscular theory, others only on the undulatory.

(7) Hypotheses should *not* be *unnecessarily multiplied.* This is required by the law of parsimony. If we can successfully explain facts by a few hypotheses, it is unnecessary and undesirable to adopt a large number.

The law of parsimony

(8) A *simple* hypothesis is preferable to a complex one. This is an aesthetic demand. Scientists believe that the world can be explained by a few simple laws. But though a simple hypothesis may sometimes explain matters clearly and definitely, it should not be preferred to a complex one at the cost of truth.

A simple hypothesis is preferable to a complex one. The law of simplicity

Proof of Hypothesis

We need not add much to what we have already stated

to explain the conditions of proof of a hypothesis, since the conditions of legitimate hypothesis are at the same time the conditions of its proof. But we may lay special stress upon one condition, *viz.*, that to *prove* a hypothesis we must be able to *deduce conclusions* from it and *compare* them with observed *facts.* If we find that the deduced conclusions agree with observed facts, and if we can also pass back to the hypothesis from these conclusions, we may assume that the hypothesis is true. Thus to prove a hypothesis we require the help both of deduction and of induction. Since conclusions deduced from a hypothesis are compared with facts observed and experimented upon, it is wrong to suppose that either deduction or observation and experiment are the only method of proof. Whewell and Jevons are exponents of the former view, Mill of the latter; but each of them is one-sided in his view. Jevons further observes that hypotheses are always problematic, and that a hypothesis is highly probable if it can disprove rival hypotheses. But mere disproof of contending hypotheses is not sufficient to establish a hypothesis. When a hypothesis is proved, it becomes a theory. The principle of gravitation, the molecular constitution of matter, etc., are now regarded as proved hypotheses.

Both deduction and induction are necessary to prove a hypothesis

Hypothesis and Abstraction

**[Every class idea involves abstraction. Thus the idea of 'crow' is not the same as the idea of this or that crow. The idea of 'blue' is not the same as the idea of 'this blue'. This abstraction involves a certain divorce from the contents of reality. But this separation of a notion from the perceived objects admits of degrees. Classification involves

less abstraction than causal determination, and this again involves less abstraction than measurement. Mathematics is the most abstract science. Mathematical notions are absolutely ideal and formal: a geometrical point, which has no magnitude, or a geometrical line, which has no breadth, has no counterpart in reality. A physicist may define a bar as absolutely rigid, but such a bar can never be presented in experience. Such conceptions are limiting conceptions, to which the things of the world only approximate. But a hypothesis can never be as abstract as a mathematical notion. It is formed to explain facts of the world, and if it be completely divorced from what we experience in the universe, it can never gain acceptance. But hypothesis which is general and which is about causal laws must always involve some abstraction, since a general notion, as we have seen, is never the same as an object of perception. Some elements which are present in perception are disregarded when the facts are explained by a general notion. Thus though hypothesis and induction involve abstraction, yet the resulting notions are never as abstract as mathematical concepts.]**

Hypotheses are never so abstract as mathematical concepts

Newton and Hypothesis.

** [Einstein once wrote that science will attain simplicity when the facts of the world are explained, by a few mathematical conceptions, which means introducing a gap between hypotheses and sense-experiences. Galileo also tried to explain sensible facts by mathematical conceptions, and was charged with having committed "the rape of reason upon the senses". Should hypothesis be divorced from facts? We have already remarked that though induction involves abstraction, it should not have

What does Newton mean by his statement, Hypotheses non fingo ?

recourse to hypotheses which are alien to facts. Newton was not inimical to the hypothetical method, though he once declared, "Hypotheses non fingo" (I do not make hypotheses). According to him, the hypotheses formed to explain facts should be allied to them, and must not be abstract notions. He was inimical to the making of metaphysical hypotheses and wild guesses. Taking advantage of this saying of Newton, some have compared hypotheses to bands of wild tribes without law and government. But Newton only urged scientists to be cautious in framing hypotheses. His attitude towards hypothesis will be clear if we observe four rules of philosophising given by him in his "Principia". Rule I is, "No more causes of natural things are to be admitted than such as are both true, and sufficient to explain the phenomena of those things". This rule is important. It requires that hypotheses should not unnecessarily be multiplied. Further it requires that hypotheses should be of verae causae, that is true causes. We have already explained the meaning of this. This rule also requires that hypotheses should be adequate. Rule II is, "Natural effects of the same kind are to be referred as far as possible to the same causes". This rule also is valuable. Effects which have the same nature may be referred to the same cause. But to do this we must analyse facts very carefully, and compare them with the conclusions rigorously deduced from a hypothesis clearly conceived. Rule III is, "Those qualities of bodies that can neither be increased nor diminished in intensity, and which are found to belong to all bodies within the reach of our experiments, are to be regarded as qualities of all bodies whatsoever". This rule as it stands cannot be accepted. In science we are not justified in extending observed qualities, even if they do not admit of variation and are found in all bodies, to unobserved cases without discovering some causal connection.

He is not inimical to all hypotheses, but only to wild guesses

His rules for philosophising

Simple enumeration is unscientific because it extends observed qualities to unobserved cases without discovering any causal connection. Rule IV is, "In experimental philosophy propositions collected by induction from phenomena are to be regarded either as accurately true or very nearly true, notwithstanding any contrary hypotheses, till other phenomena occur, by which they are made more accurate, or are rendered subject to exception". Newton here recognises that hypotheses should be rigorously tested and verified before they can be accepted in preference to other hypotheses.]**

Utility of Hypothesis

Hypothesis is a stepping stone to induction

From this consideration of hypothesis we can easily determine its utility. Hypothesis in most cases serves as the stepping stone to induction. After observing facts we often form hypotheses, and the conclusions deduced from them are again tested by observation and experiment in order to arrive at general propositions which may ultimately be ranked as inductions. Thus hypothesis is formed from experience and is verified by deduction and observation and experiment. Every induction seems once to have been a hypothesis. Though hypothesis arises from observation, it guides and directs further observation and experiment. Hypothesis binds facts together and is an attempt to explain them.

It leads to scientific explanation and to the discovery of scientific laws

Thus hypothesis, when it is proved and becomes an induction, serves the purpose of explanation. Furthermore, by colligating facts it leads to discovery. Even a working hypothesis which is afterwards rejected may direct our investigation and lead us to the framing of a scientific hypothesis.

NOTE 1: Representative fiction:—

A hypothesis may be a representative fiction. The Atomic hypothesis about the composition of matter is such a hypothesis. According to it, matter is composed of atoms which are simple, indivisible and imperceptible. A representative fiction is a notion which is legitimate but the existence of which cannot be proved by perception. That ether is the cause of light is another such representative fiction. To prove the existence of light the hypothesis of ether is legitimate though its existence cannot be proved by perception.

NOTE 2: Proof of Hypothesis:—

In our previous discussion we have laid stress upon deductive proof of hypothesis. Mill recognises the importance of deductive proof of hypothesis in induction, but he thinks that most hypotheses can be directly proved by means of his experimental methods—such as, *The method of agreement*, *The joint method*, *The method of concomitant variations*, *The method of residues*. We shall consider in a subsequent chapter how far Mill's inductive methods are effective in proving hypotheses.

CHAPTER V

SUGGESTION OF HYPOTHESIS

(Induction by Simple Enumeration and Analogy)

Introductory Remarks

Induction by Simple Enumeration and Analogy usually suggest hypotheses, but they cannot directly establish induction

We have seen what the nature of scientific hypothesis is. Hypotheses are often suggested to our mind by Induction by Simple Enumeration, and by Analogy. These processes of reasoning, however, cannot be regarded as inductive from the scientific point of view, because the conclusions they establish are more or less precarious and are not based upon the discovery of causal connection. The relation they establish between the subject of inference, and the inferred qualities, is only casual and not causal. This will become clear as we go on. We may however, observe that simple enumeration and analogy may ultimately lead to scientific induction. It is held by some logicians that simple conversion of the proposition A may also lead to the formation of hypothesis. Thus if we pass from All S is P to All P is S, which is not allowed by the rules of conversion, our conclusion may be a hypothesis, which may ultimately be accepted or rejected. It does not appear, however, that simple conversion of an A proposition has often in practice suggested a hypothesis to a scientist. We may now pass on to the consideration of Induction by Simple Enumeration and Analogy.

Induction by Simple Enumeration

"The induction which proceeds by simple enumeration is childish; its conclusions are precarious, and exposed to peril from a contradictory instance; and generally it decides on too small a number of facts, and on those only which are at hand" (Bacon).

Scientific induction and induction by simple enumeration compared

Induction by simple enumeration differs from scientific induction inasmuch as its *conclusion* is not proved, and therefore is only *probable*, whereas the conclusion of a scientific induction is proved by the discovery of a causal nexus and is therefore universal and necessary. Again, while perfect or complete induction establishes a conclusion which is merely the summary of observed instances, induction by simple enumeration provides one which is general and refers to more than the instances in the premises.

Perfect induction and induction by simple enumeration compared

We have induction by simple enumeration when the conclusion rests upon the observation of a few instances and uncontradicted experience. In scientific induction we take note of both positive and negative instances, whereas in induction by simple enumeration we take note only of positive instances and our generalisation is hasty.

Nature of simple enumeration explained and illustrated

Such generalisations as, Quinine always cures ague, Heat expands bodies, All crows are black, All swans are white, Every scarlet flower is without fragrance, When different metals are fused together, the alloy is harder than the various elements, etc., are inductions by simple enumeration.

Conclusions arrived at by simple enumeration are usually called empirical laws

Such generalisations are called *empirical* laws because they are true within the limits of present experience, but are not known to be true universally. Such empirical laws are supposed to be deducible from higher laws of universal certainty. But until they have been so deduced they are precarious, and have only a limited range of application. Thus, All swans are white is a proposition which is true only in reference to European swans, since swans of other colours have been discovered elsewhere. Again, All crows are black is true only in reference to crows within certain areas, for crows of other colours have been discovered in Australia. Similarly the proposition, Quinine cures ague is true within the range of our experience but may not be true universally. The laws of the flux and reflux of the tides in different places, the laws of weather, etc. are also empirical laws. Inductions by simple enumeration are rendered precarious not only by the spatial limits of our observations but also by temporal limitations. Swans or crows may in the course of evolution develop colours different from those which we now find.

Induction by simple enumeration rests upon counting of instances and not upon causal connection

Inductions by simple enumeration are so called because they rest upon the mere counting of observed instances, and not on an observed causal connection between the subject of inference and the inferred quality. Thus no causal connection, such as alone can justify us in passing from the known to the unknown, is known to exist between heat and expansion of bodies, between being a crow and having black colour, etc.

Mill regards the principle of causation as an empirical law. Thus he says:—"Laws of causation which are derivative and compounded of simpler laws are not only, as the nature of the case implies, less general, but even less certain, than the simpler laws from which they result, and not in the same degree to be relied on as universally true." This also is a paradox of induction. If the principle of causation itself is only an empirical law, that is, based upon uncontradicted experience, and is only probable, then how can induction, which is grounded upon the principle of causation, establish universal, necessary and real propositions? Bain however, though an empiricist, goes beyond Mill when he says that though causation is an empirical law, "yet in the case of the principle of causation the agreement is so universal, so free from contradiction, so comprehensive, that in this case humanly speaking, its efficacy cannot be doubted". We have however seen in Chapter II that the principle of causation is the postulate of induction and is presupposed in all inductions, that as such it is universal and necessary, and that in every case of causation the relation between cause and effect is reciprocal.

Can causation be regarded as an empirical law? Mill's view paradoxical

Mill's view criticised

Bain's view as to whether causation is an empirical law

It will now be easy for us to understand the logical character and value of induction by simple enumeration. Some inductions of this kind are more probable than others. If, after observing several paintings by a particular person to be excellent, we conclude that

Logical character and value of induction by simple enumeration

all his paintings are so, our argument is probable. But if we argue that since metal is a good conductor of electricity, therefore every substance is a good conductor of electricity, our conclusion is precarious. An induction by simple enumeration is overthrown whenever a contrary instance is discovered. Thus the generalisation, All crows are black, has been overthrown. An inference by simple enumeration involves the logical fallacy of the illicit minor. It can be reduced to a syllogism of the third figure.

Some such inductions are more probable than others

Thus the argument A B C D etc. are P (white), A B C D etc. are S (swans), therefore all S (all swans) is P (white), involves the fallacy of the illicit process of the minor term. S is distributed in the conclusion though it was not distributed in the minor premise. Thus the only valid conclusion that we can reach by simple enumeration must be in the form, Every S may be P, and not in the form, Every S is P.

If reduced to syllogism, simple enumeration is found to involve the fallacy of the illicit minor

Thus induction by simple enumeration may suggest a hypothesis, which may afterwards be proved by further analysis. Even when it is disproved and thrown overboard, it may lead to another hypothesis which may ultimately be found to be true.

It is useful as it suggests hypotheses which may ultimately lead to induction

Let us see how an induction by simple enumeration may lead to a scientific induction. It has been observed that all polar bears have white fur, from which we may hastily generalise that all polar animals are of white colour. This conclusion is precarious. In fact

An example to show how simple enumeration leads to scientific induction

not all polar animals are white. But after finding that some polar animals are not of white complexion, we may analyse the subject of inference and discover that those polar animals which require concealment for the preservation of life evolve white colouring. Thus when we discover the causal connection between the need of self-preservation and the possession of white colour, and establish the conclusion that all polar animals which require concealment have white colour, our conclusion becomes a scientific induction. Induction by *simple enumeration* leads us on the one hand through *analogy* to *scientific induction*, and on the other hand to *arithmetical computation* or *probability*, the nature of which we shall discuss in a subsequent chapter.

Analogy

Both Induction by Simple Enumeration and Analogy are regarded as inductive, because in such reasonings we start from observation of facts; but the conclusions they establish are not as certain as the conclusions established by inductions proper. They are therefore not scientific inductions. "To reason from one subject to another of a different kind" on the basis of *similarity*, "might be called reasoning by *analogy;* yet the inference might be such as to deserve the name of induction". Every inference rests upon some points of similarity; but while *induction* is possible only when similarity in *essential* points is known to exist between observed data, *analogy* rests upon *any* similarity whatsoever, whether in essential points or not. So Bain says, "Analogy, as different from induction, and as a distinct form of inference, supposes that two things from resembling in a number of points,

Nature of Analogy

Bain's definition of analogy

may resemble in some other point, which other point is not known to be connected with the agreeing points by a law of causation or of co-existence." If the relation is quantitative, then the argument will be mathematical. *Analogy* in *mathematics* is commonly called *proportion*. This form of analogy requires more than two terms instead of two, and is necessary. If a:b::c:d, then if a weighs twice as much as b, c will weigh twice as much as d. Aristotle recognised this form of analogy, though this is not the only or even the usual form. But suppose the distance from a to b is the same as from c to d, it does not follow that if we are to go from a to b by train, we shall have to go from c to d by train. If the relation between patient and doctor is the same as that between customer and tradesman, then it may be correctly inferred that just as a patient may be treated by different doctors, so a customer may buy from different tradesmen. The Chinese supported the despotic form of government with the argument that since the paternal form of family is despotic and good for the family, so also the paternal or despotic form of government is good for the state. But in such an argument the points of similarity may not be relevant to the question at issue, and the points of difference are ignored. In the same way it is argued that since the relation between the mother-country and the colonies is similar to the relation between mother and children, therefore just as a mother can claim support from her children, so the mother-country can legitimately claim tribute from the colonies. Here also the argument is unsound, because it ignores the points of difference which are relevant to the question. Sir Henry Maine observes: "Analogy, the most

Analogy based upon quantitative relation

Some examples of analogy requiring four terms

valuable of instruments in the maturity of jurisprudence, is the most dangerous of snares in its infancy."

All these arguments from analogy rest upon the resemblance of relation

So far we have given examples of analogy resting upon the resemblance of *relation;* but "the analogy may be *any resemblance* between two things, and not merely a resemblance of the relations in which they respectively stand to two other things; and the argument from analogy may be an argument from some degree of resemblance to a further resemblance, not an argument from the consequences of a relation in one case to its consequences in another".

The usual form of the analogical argument

The usual form of the analogical argument is:—

P has the properties X, Y, Z and M,
S has the properties X, Y, Z,
∴ S has the property M.

e.g., The student X is intelligent and industrious and he passed the examination, The student Y is also intelligent and industrious, ∴ The student Y will pass the examination. Analogy therefore is an argument "from partial identity of content to further identity of content". Lotze states the principle of analogy thus:—"Of like things under like conditions like assertions are true."

Not all analogical arguments are equally plausible. Some may suggest legitimate hypotheses, leading ultimately to scientific induction, but others may be childish. Thus if we argue that Shakespeare was a man and wrote great books, therefore you who are a man can also write great books, our

argument is childish. We often hear it said that what man has done, man can do; but this also is an analogical argument that cannot be defended. The following is another example of a futile analogical argument. To refute Galileo's discovery of the satellites of Jupiter, Francesco Sizzi argued: "There are seven windows in the head, two nostrils, two eyes, two ears and a mouth; so in the heavens there are two favourable stars, two unpropitious, two luminaries, and Mercury alone undecided and indifferent. From which and many other similar phenomena of Nature, such as the seven metals, etc., which it were tedious to enumerate, we gather that the number of planets is necessarily seven."

Some arguments from analogy are childish and futile. Examples

The following are plausible arguments from analogy: The students of the schools A, B, C, D are successful in examinations because they are of a high social status and the teachers of the schools are efficient. Therefore the students of the schools X, Y, Z, who are of the same social status and receive instruction from efficient teachers, must also be successful in examinations. The following argument is given by the philosopher Thomas Reid:—"We may observe a very great similitude between this earth which we inhabit, and the other planets, Saturn, Jupiter, Mars, Venus and Mercury. They all revolve round the sun, as the earth does, although at different distances, and in different periods. They borrow all their light from the sun, as the earth does. Several of them are known to revolve round their axis like the earth, and, by that means, must have a like succession of day and night. Some of them have moons, that serve to

Some arguments from analogy are plausible

Examples

give them light in the absence of the sun, as our moon does to us. They are all, in their motions, subject to the same law of gravitation as the earth is. From all this similitude, it is not unreasonable to think, that these planets may, like our earth, be the habitation of various orders of living creatures. There is some probability in this conclusion from analogy." The properties which are enumerated are important because they are related to one another. Further investigation however is necessary to establish or overthrow the conclusion. If air be an indispensable condition of human life, then it is necessary to investigate whether Mars etc., are surrounded by air. This analogy is no doubt suggestive.

The conditions upon which the value of analogical argument rests, as given by Mill and his followers

Mill and his followers enumerate four conditions upon which the value of an analogical argument depends. They are: (1) the greater the number and importance of the points of agreement, the more probable is the inference. A *property* is *important* if it is *relevant* to the point to be established, that is, the importance of a property is determined by the purpose in hand. A property which is important in one reference may be unimportant in another. (2) The greater the number and importance of the points of difference, the less probable is the inference. (3) The greater the number of unknown properties in the subject of our argument, the less the value of any inference from those that we do know. In the above example, we know that the two students agree in intelligence and industry, but we may not know whether they agree in other respects or not. If these unknown properties are numerous and important, then an analogical argument based upon the known properties is precarious. (4) The fewer the number of unknown properties in the subject of

our argument, the more valuable is the inference from those that we do know. If these unknown properties be unimportant and few in number, then the analogical argument from known properties may lead to valuable results.

We may however offer two criticisms of these rules which are intended to determine the value of analogical arguments. The first is that in analogy the quantity or number of known or unknown properties is not important.

Criticism of Mill's view

Secondly it is not possible to determine whether unknown properties are either important or unimportant, or whether they are many or few.

We may now conclude this topic with the remark that analogy cannot prove its conclusion, it can only lead us to the formation of a hypothesis which may prove helpful to the attainment of truth. Thus analogy may lead us to induction. Both analogy and induction may rest upon a universal connection, but whereas in analogy this connection

Induction and analogy compared

is merely suggested, in induction it is proved. Further an argument from analogy is always from the particular to the particular, but in induction we usually argue from observed particulars to a universal conclusion. Both analogy and induction, however, rest on the observation of points of similarity. But whereas the points of similarity on which induction rests are important and numerous, the points of similarity on which analogy rests are superficial, that is, unimportant and are a few in number. Induction may pass either from the particular to the general or from the particular to the particular, but analogy always passes from the particular to the particular. If we argue from the mortality of some men to the mortality of all

men or from the mortality of a particular man to the mortality of another particular man, our argument in each case is inductive, because it is based upon the knowledge of causal connection between humanity and mortality. But in the case of analogy no such causal connection is known to exist between the subject of inference and the inferred property. Thus in the above example of the two students agreeing in intelligence and industry, we do not know if there is any causal connection between these attributes and the attribute of being successful at the Examination. So it is said that while the conclusion of induction is certain that of analogy is only probable.

CHAPTER VI

MILL'S INDUCTIVE METHODS

(Establishment of Hypothesis)

The Inductive Methods of Mill are also called *Inductive Canons*, *Experimental Methods* or *Experimental Canons*. Mill supposed that inductive logic ought to investigate rules or instruments by means of which causal connections can be established, and inductive generalisations made, just as deductive logic provides rules of syllogism to prove its conclusions. These canons or methods were supposed by Mill to be of supreme importance in inductive logic, and he was followed in his belief by many logicians. The canons were vaguely recognised by Herschel. Mill

Mill's attitude towards inductive methods

systematised them and gave their number as five. These are (1) the Method of *Agreement;* (2) the Method of *Difference;* (3) the *Joint* Method, which is also called the Joint Method of Agreement in presence and in absence, the Joint Method of Agreement and Difference, the Indirect Method of Difference, or the Double Method of Agreement; (4) The Method of *Concomitant Variations;* and (5) The Method of *Residues.* Of these the method of agreement and the method of difference are generally regarded as fundamental, while the joint method is regarded as a modification of the method of agreement or of the method of difference. The method of concomitant variations and the method of residues are generally supposed to be modifications of the method of difference. Some logicians however consider the method of concomitant variations to be in some cases a modification of the method of difference, in others of the method of agreement.

Names of the methods. The method of agreement and that of difference are fundamental, the others follow from them

According to Mill, a *cause* is an *invariable* and *unconditional antecedent* of its *effect.* His problem was to prove causal connection between phenomena by the application of these canons. Hypothesis suggests a causal relation between two events, and this supposition may often be proved directly without the help of deduction by means of these experimental canons. These are therefore called *direct* methods, and their aim is to establish causal connection between phenomena by direct observation and experiment. Now both causal situations and effect situations are complex. Some aspect of a given situation may be causally connected with some aspect of another situation which succeeds it; but along

Mill's problem

with these there are other circumstances, both antecedent and consequent, which are not relevant to the question at issue. Mill's canons aim at eliminating these irrelevant circumstances, in order to find out those which are causally connected. Suppose A is the cause of X. A does not occur alone, but is combined with B, C, D, etc. Similarly X is combined with P, Q, R. In this case, how are we to find out that A is the cause of X or that X is the effect of A? We can do it if by varying the circumstances B, C, D and P, Q, R —that is, eliminating them, we find that whenever A is, X is; and whenever A is absent, X is absent. These methods therefore are methods of *elimination*. We shall see later how far these canons can successfully eliminate irrelevant circumstances.

These methods are methods of elimination, and they aim at proving inductions by establishing causal connections between phenomena

From the definition of cause as a necessary antecedent we can deduce three principles of elimination, *viz.*, (1) whatever antecedent can be left out without prejudice to the effect, can be no part of the cause; (2) when an antecedent cannot be left out without the consequent disappearing, such antecedent must be the cause or a part of the cause; (3) an antecedent and a consequent rising and falling together in numerical concomitance are to be held as cause and effect. From the first of these principles follows Mill's method of agreement, from the second the method of difference, and from the third the method of concomitant variations. We have already mentioned that the joint method follows from the methods of

The principles of elimination are deducible from the definition of cause

The canons are deducible from the principles of elimination

agreement and difference, the method of residues from the method of difference, and the method of concomitant variations either from the method of difference or from the method of agreement. Thus we find that all Mill's canons of induction can be ultimately deduced from the principle of causation. We may now give an account of these methods one by one.

They aim at proving hypothesis directly by observation and experiment

The Method of Agreement

This method is stated by Mill as follows:—

"If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon."

The general statement of the method

This canon may be expressed by the following symbols: A B C, a b c; A D E, a d e; A F G, a f g; ∴ A is the cause of a. These symbols show that B C, D E, F G, and b c, d e, f g, vary while A and a are constant. Therefore it is concluded that A is the cause of a. In these three instances of the phenomenon under investigation, say a, the only common circumstance is A; therefore it may be presumed that a and A are related in the way of causation, that is, one is either the cause or the effect of the other. Bain gives the following concrete illustration of the method. Bodies assume crystalline structure under various circumstances. But while other circumstances vary, one is constant, *viz.*, the solidification of a substance from a liquid state. Therefore it is concluded that the solidification of a substance from a liquid state is the cause or the invariable

Symbolic statement of the method

antecedent of its crystallisation. Though countries vary in different characteristics, such as temperature, vegetation, fertility of the soil, political and economic conditions, etc., it is found that whenever a particular type of mosquito is prevalent, malaria is also prevalent. Therefore it may be concluded that these mosquitoes are causally connected with malarial fever. Various races and peoples which differ in many respects have some institutions in common. These have been accepted because they have been found to be socially useful. So social utility may be regarded as the reason for the existence of these common institutions.

Defects of this method

This method is a method of observation and therefore it suffers from various defects. It cannot make use of analysis to any large extent. It can strengthen a hypothesis which suggests a causal connection, but cannot prove it. "The method of agreement resembles simple enumeration in its reliance on number of instances, but it differs from it in the stress laid on variety in the accompanying circumstances." The conclusion established by this method can never claim certainty, but its value may be strengthened if a large number of instances are examined and if the instances are exhaustively representative. But this demand is never fulfilled by Nature.

It requires a large number of instances

It may be vitiated by plurality of causes

Further this method, like every direct method, is peculiarly liable to be vitiated by plurality of causes. In the above symbolic example, a in the first instance may be caused by either B or C, in the second instance by D or E, and in the third by F or G. Mill recognises this defect of this method, and says that it is capable of establishing an *invariable*, but

not an *unconditional*, relation between an antecedent and a consequent.

It fails to distinguish whether a particular relation is one of mere invariable succession or of causation

Again this method fails to distinguish whether a particular relation is one of mere invariable succession or of causation. The co-effects of the same cause may be supposed to be causally related if we rely upon this method alone. Thus day and night succeed each other at different seasons and in other varying circumstances. If we rely upon this method alone, we may suppose that they are causally related, though they are really the co-effects of the same cause, *viz.*, the rotation of the earth in the course of its revolution round the sun.

The Method of Difference

General statement of the method

Mill thinks that the method of difference, wherever it can be applied, supplements the deficiencies of the method of agreement. He gives the following statement of the method:- 'If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former: the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon." This may symbolically be stated thus:—

Symbolic example

A B C — a b c;
B C — b c;
∴ A is the cause of a.

We here see that the instances differ only in one

respect. In one instance A a are present, while in the other they are absent. But B C, b c are constant. Therefore it is concluded that A is the cause of a.

This method is a method of experiment, and if it can be rightly applied, it is believed that it can establish causal connection and prove a hypothesis. This method is applied frequently in everyday life. A man is thirsty, he drinks water, and his thirst is quenched. In this case the drinking of water is regarded as the cause of the quenching of thirst. A man in perfect health is shot through the heart and subsequently dies. From this it may be concluded that being shot through the heart is the cause of his death. A man, when he lives in a particular place, say Calcutta, suffers from a certain disease, but when he moves to another place, he does not suffer from the disease. From this it may be concluded that some circumstance in Calcutta is the cause of his disease. By this method "a man attributes a bruised elbow to a fall, a sudden draught to the just opened door, the blotting of a landscape from view to an uprolling mist". Similarly by introducing oxygen into or removing it from a body we may prove that oxygen helps combustion. Again by adding sodium to diluted sulphuric acid it may be shown that sulphuric acid causes the release of hydrogen.

Concrete examples

If the requirements of this method can be rigorously fulfilled, it is generally successful in establishing causal connection. This method is a simple one, for it requires only two instances, in one of which the phenomenon in question is present, and in the other is absent, its presence being followed or preceded by the presence,

Utility and limitations of this method

and its absence by the absence, of another phenomenon. But the difficulty of applying this method is recognised by Mill. He says: "It is very seldom that Nature affords two instances of which we can be assured that they stand in this precise relation to one another". Further, since this is a method of experiment, great knowledge and practical skill are necessary in applying it. Again, to attain the desired result from the application of this method, "too great an interval must not elapse between the introduction of the supposed cause and the noting of the effect." A doctor may prescribe quinine to a patient suffering from malaria. The patient takes quinine, but the fever does not cease at once. Suppose after an interval of two or three days the fever ceases. Can we definitely ascertain that the taking of quinine was the cause of the cure? During the interval something else might have taken place and that might have been the cause of the cure.

The Joint Method

The method of difference, as we have seen, requires special and determinate circumstances if it is to attain its object. But since the application of this method is not always possible, we may attain the same result by the application of the *joint method*, the conditions of which may be fulfilled with less difficulty. This method is also called the double method of agreement, the method of agreement in presence and in absence, etc. It requires two sets of instances, in one of which the phenomenon in question is present and in the other absent, its presence being preceded or followed by the presence, and its absence by the absence, of another phenomenon, while other circumstances vary. The two phenomena which agree both in pre-

The need of this method

sence and in absence amidst variation of other phenomena, are then supposed to be causally connected. Mill states the method thus:—

General statement

"If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon." This may be stated symbolically:—

A B C — a b c,	B C — b c;
A D E — a d e,	D E — d e;
A F G — a f g,	F G — f g;

∴ A is the cause of a.

Here we have two sets of instances; in one set A and a are both present and in the other they are both absent, while other circumstances vary. So it is concluded that A and a are causally connected.

This method is supposed by some to be a modification of the method of agreement, since in one set of instances A and a agree in being present, while in the other set A and a agree in being absent. Others hold that it is a modification of the method of difference as well as of agreement. The fact that A and a disappear together in the second or negative set of instances is held to constitute this method as a modification of the method of difference; while the positive set, and the varying of other circumstances in spite of the agreement of A and a in presence and absence, are supposed

to affiliate it to the canon of agreement. It seems to us that the joint method may properly be regarded as a modification of the method of agreement, since the circumstances other than A and a are varying.

The nature of the method

Suppose a man suffers from sleeplessness whenever he takes coffee at night, but not, when he refrains from taking coffee at night, though other circumstances vary; we may then conclude that the taking of coffee is the cause of his sleeplessness. If it is found that whenever mosquitoes of a particular type are present malaria is also present, and whenever they are absent malaria is also absent, though other circumstances vary, we may conclude that the presence of these mosquitoes is the cause of malarial fever.

This method is also used in the ordinary affairs of life. There is no doubt that it largely remedies the defects of the method of agreement, and is not usually liable to error from plurality of causes if the conditions of the method are properly fulfilled. This method, therefore, is generally capable of establishing causal connection, like the method of difference. Its conditions, though stringent, are not as stringent as the conditions of the methods of difference.

Utility and defects of the method

The Method of Concomitant Variations

This method is applicable in cases where perfect elimination of the cause is not possible, as in the case of "the laws of those permanent causes, or indestructible natural agents, which it is impossible either to exclude

The nature of the method

or to isolate''. Thus heat, gravity, friction etc. cannot be completely eliminated from a body, but they may be increased or diminished in quantity, and the effects resulting from such increase or decrease can be observed. This method rests upon the belief that the energy of the cause is equal to the energy of the effect, that is, that increase or decrease in the one must be followed by a proportionate change in the other. By means of this method quantitative relations between the cause and the effect can be established. Mill states the method thus:—

Statement of the method

"Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation." This method is expressed generally by the

Symbolic statement

$$\begin{aligned} &A\ B\ C - a\ b\ c\ ; \\ &A_1\ B\ C - a_1\ b\ c\ ; \\ &A_2\ B\ C - a_2\ b\ c\ ; \\ &\therefore A \text{ is the cause of } a\ ; \end{aligned}$$

but it is also expressed by the symbols—

$$\begin{aligned} &A\ B\ C - a\ b\ c\ ; \\ &A_1\ D\ E - a_1\ d\ e\ ; \\ &A_2\ F\ G - a_2\ f\ g\ ; \\ &\therefore A \text{ is the cause of } a. \end{aligned}$$

But if the method of concomitant variations is regarded as a modification of the method of difference, the first set of symbols is preferable to the second, because in it A and a vary in degree while other circumstances remain the same. In the second set, not only A and a, but other circumstances

also, vary and the method then appears as a modification of the method of agreement. In this second form it is incapable of establishing causal connection with any degree of certainty. Both sets of symbols show that the variation of a corresponds with the variation of A, and this variation is in degree. It is therefore presumed that A is the cause of a. When the variation between A and a is in proportion, we can be almost sure of the result, but the variation may not be in proportion, *e.g.* A—a, A_2—a_3, A_3—a_7. In such a case A is not the entire cause of a, but is only a part of the cause. The variation between A and a may be either in the same direction or in opposite directions. The increase of A may be followed either by the proportionate increase or by the proportionate decrease of a.

The first set of symbols is preferable to the second as the former is less open to risk

Further elucidation of the nature of the method

Other things being equal, with the increase of heat mercury in the thermometer rises proportionately: therefore it is concluded that heat is the cause of the rise of mercury in the thermometer. Similarly, the more a body is heated, the more does it expand; therefore heat is the cause of expansion. These concomitant variations can be represented by graphs, one line showing the rise of temperature, and other line showing either the gradual rise of mercury or the gradual increase of expansion. Other things being equal, if with the gradual increase of poverty crimes such as theft, robbery etc., gradually increase, it may be presumed that poverty is the cause of such crimes. Again, if it be found that with the increase of the

Concrete examples

activity of the police crimes diminish proportionately, then it may be concluded that the increase of police activity is the cause of the diminution of crimes. A French doctor showed that with the increase and decrease of the production of vintage, crimes also increased or diminished proportionately; from this it was concluded that the production of vintage is causally connected with crimes. Statistics of prices reveal causal connection between price and other factors by the method of concomitant variations. Thus we may statistically represent the relations between production and price, or between price and duty imposed on an article, in a graduated series by graphic representation; that is, we can show by graphs how prices fall with the increase of production and how they rise with the increase of duty.

Further, by arranging certain things of the same class in a graduated series, with reference to a certain property, we may show how with the differences in degree of that property, certain other properties also vary in degree. Thus if, after arranging human brains in a graduated series according to weight, it is found that the greater the weight of a man's brain the greater is his intelligence, it may be presumed that there is a causal connection between the weight of a human brain and intelligence.

We must however note that there are limits to the application of the method of concomitant variations. Water proportionately contracts with the cooling of temperature up to a certain point, but not beyond that point. It continues to contract as the temperature falls to 39° F., but below that point it begins to expand, and the further the temperature is lowered, the greater the expansion that takes place. Again, with the gradual increase of the application of capital and labour to production, there may be either increasing,

diminishing or constant returns. The law either of increasing returns or of diminishing returns or of constant returns holds good up to a certain point but not beyond.

Utility and defects of this method

This method very often helps us to form hypotheses, and may ultimately lead to discovery. It cannot always prove a causal connection, and is more useful as a method of discovery than as a method of proof. It may be vitiated by plurality of causes, especially when the gradual variation of a particular antecedent, say A, and a particular consequent, say a, is accompanied by variation in other circumstances also. Further the concomitant variation of co-effects of the same cause may wrongly suggest that there is a causal connection between them.

Mill's attitude towards the four methods already discussed

The four methods which we have now explained are regarded by Mill as the only inductive or experimental methods. He says that these four methods, with such assistance as can be obtained from deduction, "compose the available resources of the human mind for ascertaining the laws of the succession of phenomena".

The Method of Residues

General nature of the method

The fifth method is called the Method of Residues. When previous investigation has established causal connection between certain consequents of a complex sequence, by applying this method we can then prove that the remaining antecedent (or antecedents) is (or

are) the cause of the remaining consequent (or consequents). If the result previously obtained is reliable, says Mill, this method can be a method of rigorous proof. In some cases where the methods of difference and concomitant variations cannot be applied, we may have recourse to this deductive method, which may be stated as follows: "Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents". A. B C —a b c; B is known to be the cause of b, C of c; therefore A is the cause of a.

The statement of the method

Symbolic expression of the method

This method can be applied only when some progress in scientific investigation has been made. Suppose I know that a piece of stone weighs 20 seers, and when another stone is placed upon it they together weigh 30 seers. Then by the application of this method we see that the residual weight of 10 seers represents the weight of the second piece of stone. It was by means of this method that Neptune was discovered. The orbit of Uranus had been calculated in accordance with the known data. When it was found that it deviated from its calculated orbit, astronomers concluded that there must be some other agent, the mode of operation of which must be responsible for this deviation. This led to the formation of the hypothesis of a hitherto undiscovered planet; afterwards by means of a powerful telescope Neptune was observed in the calculated position, and the hypothesis was proved. Lord Rayleigh and Sir William Ramsay, when they found that nitrogen obtained from air was heavier than

This is believed to be a method both of discovery and of proof

nitrogen obtained from other sources, undertook investigations to find out the cause of this difference. This led to the discovery that another gas which existed in the atmosphere was mixed up with nitrogen obtained from air, and thus made it heavier. This gas was named argon, and it was discovered by the application of the method of residues.

Controversy whether the method of agreement or the method of difference is more fundamental

There has been a controversy whether the method of agreement or the method of difference is the more fundamental. Since all the methods are methods of elimination, that is, are concerned with the removal of irrelevant circumstances by varying them, the method of difference is supposed by some to be the more fundamental, as being efficacious in eliminating what must be eliminated before causal connection can be discovered. Others, however, hold that the method of agreement is fundamental. According to them, elimination is not the main function of the methods. By means of elimination these methods try to establish agreement between some antecedent A and some consequent a of a complex causal situation. The agreement between A and a may be either in presence or in absence. So the method of agreement is the fundamental method. This controversy seems to us to be needless. If both the methods are necessary for the establishment of causal connection, then both the method of agreement and the method of difference may be regarded as equally fundamental.

Both these methods may be regarded as fundamental

SOME EXAMPLES OF SCIENTIFIC INDUCTION

Formation of Vegetable Mould

[The thesis established by Darwin is that vegetable mould is formed by the action of earthworms; in other words, earthworms prepare the soil for the growth of plants. Darwin conducted analysis by observation. If earthworms bring up earth from beneath the surface of the ground, and afterwards rain and wind spread out this earth deposited on the surface, then the small objects on the surface will gradually become buried. Darwin records many observations to establish this. In 1835 he observed a plot of land which had been poor pasture for a long time. It was swampy and thickly covered with red sand. In 1842 holes were dug and it was found that the red sand which seven years before had been on the surface now formed a distinct layer two inches beneath the surface. Should the specific character of different pieces of land be taken into account? This point was settled by observing land which differed widely from the former, and it was found that wherever earthworms were present the result was the same. Darwin recorded that chalk formations extended all round his house in Kent. The surface of this area was irregular and full of cavities. In course of time a bed consisting of flints and other insoluble materials, carried in course of the dissolution of the chalk by rain, was formed, which was red, and 6 to 14 feet in thickness. "Over the red clay, wherever the land has long remained as pasture, there is a layer, a few inches in thickness, of dark-coloured vegetable mould." Darwin even spread chalk on the surface of the land to see the result after a lapse of time. In such a case observation partook of the nature of experiment. After 29 years it was found that the chalk had sunk 7 inches underneath the surface. This was certainly due to the action of the earthworms. Heavy and light bodies were found to sink to the same depth within the same time. So the specific gravity of the objects did not affect their rate of

sinking. Darwin observed many cases to establish his thesis. The total evidence showed that a connection does exist between the existence of earthworms and the formation of vegetable mould. It was found that boulders of huge dimensions did not sink beneath the surface. This was due to the fact that the earth beneath the boulders was kept dry, and such earth would not be inhabited by earthworms, and so the boulders did not sink into the ground. In this way negative instances were observed, and it was found that where there were no worms there was no sinking of objects. Many negative instances were recorded by Darwin. "Both positive and negative instances, then, go to support the universal propositions, 'If there are worms, there is vegetable mould' and 'If there is vegetable mould, there are worms'."

The Laws of the Pendulum

In establishing the laws of the pendulum, experiment was resorted to and exact results were obtained. "Galileo, by experimenting on balls suspended by threads of different lengths, discovered that the time of oscillation depends upon the length, and is proportional to its square root." Newton demonstrated this conclusion more exactly. The temperature, which was a residual phenomenon and which was the same for each pendulum used throughout the experiment, could not affect the result. Among the remaining residual phenomena were gravity, the resistance of the air, and the substance of which the pendulums were made. These also were analysed and eliminated. "Newton made two pendulums of equal spherical wooden boxes suspended by strings of equal lengths. In the centre of these boxes were placed equal weights of various metals" such as gold, silver, lead, etc. in turn. By securing similar shape and equal size and weight, the influence of the air was eliminated. "Newton found that the pendulums thus made oscillated for a length of time with equal oscillations." It was thus shown that the materials of which the pendulums were made did not make

any difference to their oscillation. It was established that all substances were equally influenced by gravity. "Length, which was already known to be operative in the time of oscillation, was thus proved to be the only operative condition determining the time of oscillation at any one place."

Source of Power in the Voltaic Pile

Faraday proved experimentally that the source of power in a voltaic pile is due to chemical action. He undertook many experiments to establish the proposition, "If chemical action, then an electric current", and its reciprocal, "If a current, then chemical action", *i.e.*, "If there is no chemical action, there is no current". He examined both positive and negative instances. By many experiments "he established the positive connection, and showed that chemical action is both efficacious in producing a current, and sufficient by itself, and without any contact, to do so". "When tin was associated with platinum, gold, or, I may say, any other metal which is chemically inactive in the solution of the sulphuret (of potassium), a strong electric current was produced", and as the chemical action decreased and finally ceased in consequence of the formation on the tin of an "insoluble, investing, non-conducting sulphuret of that metal", the electric current diminished and finally ceased also. "If two pieces of silver be associated in strong muriatic acid, first the one will be positive and then the other; and changes in the direction of the current will not be slow, as if by a gradual action, but exceedingly sharp and sudden." Numerous negative experiments were also performed. It was shown that where there was no chemical action, there was no electric current. "Two plates of iron and platinum are placed parallel, but separated by a drop of strong nitric acid at each extremity. Whilst in this state no current is produced........ but if a drop of water be added........chemical action commences, and a powerful current is produced, though without metallic or other additional contact." In this way by various

experiments Faraday established his thesis that the source of power in a voltaic pile is due to chemical action.

Mode of Action in the Sense of Smell

The thesis here proved is that "the mode of action in smelling is oxidation". Graham recorded that "the sweet odours are due to hydrocarbons, as the ethers, alcohol, and the aromatic perfumes". "All these substances are highly oxidizable at common temperatures, being speedily decomposed in the air." Sulphuretted hydrogen, which has a bad smell, is readily oxidized. The hydrogen compound, as it is decomposed, produces a smell. These are positive instances. To cite negative instances: marsh gas (carburetted hydrogen) is not oxidized at common temperatures, and therefore is without smell. "Again, hydrogen itself, if obtained in purity, has no smell, and it does not combine with oxygen at the usual temperature of the air." Further, "If oxygen is excluded from the cavities of the nose, there is no smell. Also a current of carbonic acid arrests the odour", as it is supposed to be hostile to oxidation. By such observations both of positive and negative instances the original thesis, "the mode of action in smelling is oxidation", was established.

Origin of Beauty of Flowers

Darwin advanced the thesis that "flowers have been rendered conspicuous in contrast with the green leaves, and in consequence at the same time beautiful, so that they may be easily observed by insects". The generalisation thus made is not absolutely certain, because here analysis is not perfect. Darwin argued that "when a flower is fertilised by the wind it never has a gaily-coloured corolla. Several plants habitually produce two kinds of flowers; one kind open and coloured so as to attract insects; the other closed, not coloured, destitute of nectar and never visited by insects".

Wallace strengthened Darwin's contention. He writes that "not only have we reason to believe that most of these wind-fertilised flowers are degraded forms of flowers which have once been insect-fertilised, but we have abundant evidence that whenever insect agency becomes comparatively ineffective, the colours of the flowers become less bright, their size and beauty diminish, till they are reduced to such small, greenish, inconspicuous flowers as those of the rupture-wort, the knot-grass, or the cryptogamic flowers of the violet. There is good reason to believe therefore, not only that flowers have been developed in order to attract insects to aid in their fertilization, but that, having been once produced, in however great profusion, if the insect races were all to become extinct, flowers (in the temperate zones at all events) would soon dwindle away, and that ultimately all floral beauty would vanish from the earth." Both positive and negative instances are taken into account by Darwin and Wallace, and the reciprocal proposition, "If flowers are insect-fertilized, they are beautiful" is established. Such cases do not admit of experiment, and we have to depend upon simple observation. So here we cannot establish a relation with quantitative exactness.] **

(For further examples students are referred to Mill, Bain, Welton, Welton and Monahan).

CHAPTER VII

MILL'S DEDUCTIVE METHOD

(Establishment of Hypothesis, continued)

Mill recognised that his canons were not always sufficient to establish causal laws, and he perceived that in some cases inductive generalisations could be proved only when induction was aided by deduction. He was not however clear in his view of the necessity and the range of application of deduction in induction. We have already stated that every inductive generalisation has to be tested and verified by deduction and comparison with facts before it can be finally accepted. Bain, though he generally was at one with Mill, differed from him in recognising that the range of the application of deduction in induction was not as narrow as Mill supposed. Modern logicians agree in this respect with Bain, and definitely hold that in *every case* of *induction* the aid of *deduction* is *indispensable* to prove the inductive generalisation.

Mill thinks that the range of application of deductive method in induction is limited

The view of Bain and modern logicians

Mill holds that in those cases in which laws of simple effects have to be established, inductive canons are sufficient to prove them. According to him, in the case of heteropathic intermixture of effects also, the law of the complex effects cannot be deduced from the laws of their separate

causes. Thus the law of water cannot be deduced from the laws of its separate causes such as hydrogen and oxygen. Here we should note that the production of water depends not merely upon hydrogen and oxygen, but also upon their combination in a definite proportion. Miss Stebbing observes that in some cases the only reason why deduction is not applicable is because our knowledge regarding such complex effects is limited, not because of any intrinsic limitation of deduction itself.

Mill however thinks that in the case of homogeneous intermixture of effects, that is, in the case of composition of causes, induction requires the aid of deduction. Thus the light produced by two candles is the sum of the lights produced by each of them. Now if we know what quantity of light each of them produces, we can by summing the two quantities find out the joint effect. This is a case of calculation or rather deduction. Similarly if two forces produce a joint effect, then the complex effect is nothing but the sum of the separate effects of these forces. To determine the laws of such complex effects three steps are necessary, *viz.* (1) induction, (2) ratiocination or deduction, and (3) verification. So Mill writes: "The mode of investigation which, from the proved inapplicability of direct methods of observation and experiment, remains to us as the main source of the knowledge we possess or can acquire respecting the conditions and laws of recurrence of the more complex phenomena, is called, in its most general expression, the *Deductive* Method, and consists of *three operations*—the first, one of direct *induction*, the second, of *ratiocination*, the third, of *verification*." In the preceding chapter we saw how the method of residues requires the aid of deduction. It consists in finding out the cause of a resi-

Three steps involved in deductive method

A distinction between deduction required by the method of residues and that required by the deductive method

dual phenomenon when the cause of a part of the total phenomenon is already known. Miss Stebbing illustrates the difference between the deduction employed by the method of residues and the nature of the deductive method as Mill conceives of it by a simple example. "If it be known that a cardboard box weighs 3 oz. and a quantity of chocolate inside the box weighs 1 lb., then the compound effect of these two quantities is 1 lb. 3 oz. Conversely, given that the total weight is 1 lb. 3 oz. and it is known that the box weighs 3 oz., then the weight of the chocolate can be deduced by subtraction." Mill regards the former alone as an instance of the deductive method, and the latter as a case for the method of residues.

"The problem of the Deductive Method is to find the law of an effect from the laws of the different tendencies of which it is the joint result" (Mill).

The necessity of induction in deductive method explained after Mill

Complex laws of human action can be deduced from simpler laws, but these simpler laws, Mill thinks, have been arrived at by induction. So the application of deduction to determine the laws of complex effects is possible only after the simpler laws from which they are deduced have been arrived at by induction. Thus *induction* is the *first step* in the *deductive method*. Mill therefore writes, "To ascertain, then, the laws of each separate cause which take as share in producing the effect is the first desideratum of the Deductive Method". The laws of the tendencies of separate causes should be inductively determined before we can deduce from them a complex effect. Thus to deduce successfully the phenomenon of life

from the mechanical and chemical laws of the solid and fluid substances composing the organised body and the medium in which it subsists, it is necessary to determine these chemical and mechanical laws by means of induction. But Mill recognises that sometimes, as in the case of physiological phenomena, the laws of the tendencies of separate causes cannot be inductively determined. He thinks that only when such simple laws can be inductively determined, can the application of the deductive method to determine the laws of complex effects achieve brilliant results. To accomplish the first step of deductive method, that is, to arrive by means of induction at a general conclusion from which deductive reasoning starts, we must observe a large number of instances if possible; observe a few instances with special care; and seek the general laws of phenomena in the cases where they are least complicated.

The second step of the deductive method, viz. ratiocination, can be resorted to only when the simple causal laws have been previously ascertained by induction

"When the laws of the causes have been ascertained, and the first stage of the great logical operation now under discussion satisfactorily accomplished, the *second* part follows, that of determining from the laws of the causes what effect any given combination of those causes will produce" (Mill). This means that when the *simple* causal laws have been first ascertained by *induction*, we can then *deductively* ascertain the law of the *complex* effect. This is true in the case of most deductions, whether the effect is simple or complex, because the general proposition with which ratiocination starts is usually provided by induction. Advanced truths of mathematics are deduced from simpler mathematical laws. To give a simple example, the law of multiplication can be

deduced from the laws of addition. Calculation is often difficult, and to be successful we must know the numerical law of each of the causes. The laws of the tides can be deductively explained by the combined action of the sun and the moon. The motion of a projectile, say the range and velocity of a cannon-ball, can be ascertained deductively by calculating the force of the charge, the angle of elevation, the density of the air, and the strength and direction of the wind, which combine to determine the effect in question. Deductive Method, again, is resorted to very often in determining the laws of astronomical phenomena.

The third step of deductive method, viz. verification

The *third* requisite of the deductive method is *verification*. The result calculated by deduction has to be verified by *comparison* with *actual facts*. Suppose we calculate from the laws of the movements of the heavenly bodies that an eclipse should take place on such a day at such a time: then if we find that at the predicted time the eclipse occurs, our deduction is verified. In astronomy, where deduction has gained its greatest triumphs, verification has also been most thoroughly worked. Many observers are now at work throughout the world, to verify astronomical predictions arrived at by deduction. We may calculate by deduction the motion and range of a cannon-ball from gravity, projectile force and the resistance of air, and this can be verified by observing the flight of an actual cannon-ball under the requisite conditions.

Our first *criticism* of Mill in this matter is that he unduly narrows the range of the application of deduction and induction. He distinguishes sharply between the methods required for dealing with combined effects and those required for dealing with simple effects.

He thinks that the aid of induction alone is necessary for the latter, that of deduction only for the former. But this distinction is arbitrary. No inductive generalisation can attain certainty without the aid of deduction; in other words, *every inductive generalisation*, whether it is a simple causal law or a complex one, remains *hypothesis until* it is *deductively verified*. Bain therefore rightly remarks that "It is desirable at every stage to carry out inductive laws into their deductive applications". Most deductive reasonings start with inductive generalisations which are ultimately proved by deduction and verification. So not only in the case of complex but also in the case of simple effects, deductive verification is necessary to prove inductive generalisations. We have already seen this in the case of complex effects. But deduction is also necessary for the simple extension of an inductive law to a new case. Thus having, by observing solids and liquids, established the general law that 'All matter gravitates', we may extend it to gases or air. This extension of induction to a new case by means of deduction has to be verified. If we argue from All material bodies gravitate, that gases and air gravitate, we have to verify the conclusion by observing whether they have weight. If we find after observation and experiment that gases and air have weight, then our conclusion is verified. Galileo threw two cannon-balls, one of which weighed 100 lbs. and the other 1 lb., from the tower of Pisa, to see whether they would reach the ground at the same time, and he found that approximately they did so. The method of investigation which Galileo followed is regarded even now as the correct scientific method. His method was, "first to formulate *provisional hypotheses* from which he reasoned *deductively* in order to ascertain whether they led to contradictory conclusions; then to *test*

Criticism of Mill's view of deductive method

these hypotheses by showing that the *experimental facts* were in conformity with them''. We may conclude this topic by quoting Bain:—"Combined Induction and Deduction expresses the full force of scientific method for resolving the greatest complications. Induction alone, and Deduction alone, are equally incompetent to the great problems even of the inorganic world; still more so with Life, Mind and Society''.

Note:

A distinction is sometimes drawn between *Direct Deductive Method or Physical Method, Inverse Deductive Method or Historical Method,* and *Geometrical Method.* Of these by far the most important is the *direct* deductive method or physical method. In the previous pages we have given a full account of this method, and it is usually called the deductive method. The name physical method has been given to it because it is applied to explain physical and mechanical phenomena such as the path of a projectile, the rise of water in a common pump, etc. By the application of this method we can explain why water can be pumped up to 33 ft. at sea level, and why it cannot be pumped up to that height when the level from which the water is pumped is higher, say, from the top of a mountain. We have already enumerated and explained the three steps which it involves, *viz.*, induction, which determines the causes of a complex effect, ratiocination, and verification. *Inverse* deductive method or historical method is applied in historical and social sciences. It is called the inverse deductive method because in it the observation of the result is followed by induction and ratiocination. To explain a revolution or an industrial revival we start with the observation of certain phenomena, and we afterwards try to formulate the laws of

their occurrence by inductive method, and then again try to explain such phenomena by ratiocination. The *geometrical* method is that which requires the help of deduction alone and no help of induction, as in geometrical demonstration. This method is also applied to some extent in Politics and Economics and such other sciences. Thus in Politics many conclusions have been deduced from the conception of the nature of right, of duty, etc. From the conception of *laissez faire* conclusions have been deduced both in Politics and in Economics. We must note that this distinction is not very scientific, because physical sciences do require, in addition to direct deductive method, inverse deductive method and geometrical method as well. Similarly historical and social sciences do often employ physical and geometrical methods. Even geometrical sciences cannot dispense with observation altogether. (For further elucidation of this subject, students are referred to Book V, ch. III).

CHAPTER VIII

PROBABILITY

The General Nature of Probability

"Probability has reference partly to our ignorance, partly to our knowledge" (Laplace). We believe that this world is uniform, and that every event can be explained by some causal law. But since our knowledge is limited, there are many events and coincidences of which we cannot give any causal explanation; and in such cases we attribute them to change. If there are several possible causes for a certain event, we often fail to determine which one of them is really responsible for the occurrence of the event.

The meaning of chance

When this is so, the coincidence of the event with one of these possibilities rather than with another is said to be due to chance. "A chance coincidence is one where there is no implied construction of cause and effect." But of the possibilities with which the events may be associated, one may be found to be more frequently associated with it than others. When this happens, we say that the *probability* of the event's being due to that phenomenon is greater than that of its being due to any other phenomenon.

The meaning of probability

Thus, suppose rain is associated more frequently with a westerly wind than with an easterly wind, then the probability of its being due to the westerly wind is greater than that of its being due to the easterly wind. Probability is a form of inference which is based upon

observation of facts, and aims at providing a mathematical estimate of the likelihood that each of a number of different possible alternatives should give rise to a certain event. It is concerned with the *calculation of chances;* that is, it tries to determine quantitatively what is the chance that each of a number of different alternative possibilities will produce a given event. If out of every five days three are dry and two rainy, then the probability of a particular day's being dry rather than wet is 3:5. If a basket contains two black balls and three white balls, then the probability of drawing a black ball rather than a white ball is 2:5 or 2/5th; that is, if there are five draws, we may expect to draw a black ball twice while we may expect to draw a white ball three times. Again, the probability of a coin showing heads when tossed is $\frac{1}{2}$ or 1:2; that is, out of every two throws we may expect to have heads once. If we throw the coin fifty times, we may expect that it will give us heads 25 times. Similarly if a die is thrown, the probability of its turning up the ace is 1/6 or 1:6; out of every six throws we may expect that it will give us ace once. If we throw it 36 times we may expect that it will show each of its six sides six times. Thus *probability* is an attempt to provide a *quantitative determination* of events by *eliminating chance.* There is no place for chance in logic, and probability, which is a logical doctrine, aims at giving a rational explanation of the world by eliminating chance and introducing mathematical calculation. But the result

Probability is concerned with the calculation of chances

Some simple examples of the calculation of probability

It aims at providing a rational account of the world by eliminating chance and introducing mathematical calculation

attained by probability, as the name implies, is only approximately true, that is, probability cannot provide certainty. Thus if we throw a die 36 times, we may not actually get ace six times, although we may expect it; for various conditions determine the result, such as throwing in a particular way, the resistance of the air, gravity and so on. Similarly if a coin is tossed 20 times it need not come up heads 10 times, since a variety of conditions determines the result. Thus the conclusion established by probability is only roughly true.

Probability can never provide a conclusion which is certain

"Probability expresses a state of the mind, and also a situation among objective facts. As a state of the mind, it is a grade or a variety of Belief." Probability is mid-way between certainty and impossibility. It is not as certain as induction, because in it inference is not based upon the discovery of causal connection. But the result obtained by probability is not impossible, since probability rests upon observation of facts. When an actuary calculates the average longevity of men, the average death rate per thousand, or the rate of death by fire, by shipwreck, by disease and so on, he has to arrive at a conclusion after careful examination of the statistics of life and death provided by different nations. His conclusion does not rest upon subjective feeling. Again, when a financier prepares a budget, he arrives at certain figures of income and expenditure by means of probability. If his budget is to correspond with actual facts, that is, is to be approximately

Probability is both subjective and objective

It rests upon observation of facts

true, then he will have to observe with great care the data, such as sources of income, items of expenditure, conditions of trade, conditions of harvest, etc., upon which the preparation of the budget rests. In our practical life we very often act in accordance with expectations which rest upon conclusions arrived at by probability.

The calculation of probability often determines the activities of daily life

Though it is true that probability is *not merely subjective*, yet it is also undeniable that the conclusion arrived at by probability is true only of the *average*. It is akin to induction in that it is guided by observation, but it differs from induction because probability can only provide us with a conclusion which is approximately true, and not as certain as the conclusion of induction. When we conclude by the calculation of probability that five Indians in every thousand live to the age of 70 years, our conclusion is likely to be true if a very large number of instances are taken into consideration. Though in every separate thousand we may not find five men living up to that age, yet if we observe 100,000 cases we may expect to find five hundred men living to the age of 70. Since averages calculated by probability are true in the long run, it is possible for sciences, such as the system of life insurance, to rest upon probability. Many conclusions in economics are true not absolutely but on the whole and in the long run. Such conclusions are arrived at by probability. There is one justification for regarding probability as subjective. Conclusions arrived at by probability being only approximately true, we may only expect them to

Calculations of probability are true of the average and in the long run

Certain sciences are based upon probability

happen, but can never be sure that they will happen. Thus when we say that if a coin is tossed 50 times it will give us heads 25 times, our conclusion represents a degree of belief which is below certainty. When an average is arrived at by probability, individuals very often deviate from the average, owing to individual peculiarities. If the students of a class are moderately intelligent on the average, it does not follow that no one of them will be highly intelligent and no one will be foolish. But the exceptions are rarer than the average. The statement only points to the fact that most students will approximate to the average, though there may be one or two exceptions. If the average of the longevity of a people be 50 years, then most men will live to that age, though there may be some exceptions.

Probability is regarded as subjective because it rests upon expectation

Particular instances deviate from the average owing to individual peculiarities

Thus when by probability we arrive at an average which is true on the whole and in the long run, we must make allowance for individual peculiarities or personal equations, in consequence of which individual cases are found to deviate from the rule. But when we say that induction itself is probable, we use the term probable in a different sense. Inductive conclusions are said to be probable because they are not absolutely certain, and this is due to the fact that human knowledge is limited and not perfect. Conclusions given by probability are only approximately true, whereas inductive conclusions are believed to be true so far as human knowledge is able to determine.

Inductions need not be called probable

Quantitative Determination

We have seen that probability is a mathematical doctrine and its calculations fall short of certainty. Sciences as they become more advanced become more capable of quantitative determinations. When we can measure our observations, we can establish quantitative laws. "Numerical precision is the very soul of science." Such sciences as astronomy and physics enable us to establish conclusions which are exact and numerically precise. We can calculate at what time an eclipse will take place or a comet will appear in the sky. The physical law of gravitation was established by Newton when he found that by means of it he could mathematically calculate relations between material objects. But since our sense-impressions often delude us, our calculations are not always quite accurate. The shape and size of the same object seem to vary with its distance from, and position relative to, the observer. Again, in measuring we use instruments, which are never perfect, however delicate they may be. So Jevons says, "We may look upon the existence of error in all measurements as the normal state of things." By means of gravitation we can calculate the position of a planet at a particular time. "If it is half a second wrong, the fault is in the instrument, the observer, the clock, or the law; now the more observations are made, the more of this fault is brought home to the instrument, the observer, and the clock." But *measurement*, since it depends upon various conditions, can *never* yield *absolutely accurate* results. Even when every kind of precaution has been taken, there are always unknown conditions which give rise to error.

The nature of quantitative determination

Measurement is never exact. To eliminate discrepancies due to unknown conditions we have recourse to probability

Probability is an attempt to eliminate these chance errors. In other words, "To eliminate these we must—as in all cases where the conditions of a phenomenon are unknown—have recourse to the theory of probability for guidance."

Logical Basis of Probability

Though scientists believe that the universe is rational and that everything can be explained by causal laws, yet owing to the imperfection of human knowledge many events appear to men to be due to chance. Still, in spite of our imperfect knowledge, we are called upon to explain everything and to act according to the best of our understanding. The province of the theory of probability is to explain those occurrences which appear to us to be casual. The calculation of probability therefore starts from the combination of knowledge and ignorance. Improbability is not opposed to probability. Improbability only implies a low degree of probability. When we say that it is improbable that it will rain to-day, we do not mean that raining is impossible to-day, but only that from what we know about the conditions of weather it is not likely that it will rain to-day. The *logical basis* of *probability* is a *disjunctive judgement*, or a combination of such judgements, coupled with deductive reasoning. The disjunctive judgement from which the calculation of probability starts must have exclusive, definite and exhaustive alternatives of equal value. "Our whole data are knowledge of the number of equivalent possible cases, and the absolute absence of any ground for preferring one rather

The province of probability. The meaning of probability and improbability

Disjunctive judgement and deduction are the logical bases of probability

than the others." If a basket contains three balls, one black and two white, then there is no doubt that a particular drawing will give either a black or a white ball; but probability aims at determining exactly, that is, with numerical precision, what the result will be. In the above example, the disjunctive proposition with which probability can start should be in the form: X is either A, or B, or C, X standing for a particular drawing and A for the black ball, B for one of the white balls, and C for the other white ball. In this disjunctive proposition we find that the alternatives are of equal value and are exhaustive and exclusive. There being three alternatives only, the probability of drawing a black ball rather than a white ball is 1/3 or 1 : 3, and that of drawing a white ball is 2/3 or 2 : 3. We also find that the calculation which is combined with the disjunction in probability is deductive. "The theory of probability is applicable to the credibility of testimony, as well as to the prediction of a future occurrence." To calculate probability we have to resort to the mathematical doctrine of permutation and combination.

Rules for the Calculation of Probability

The rule for calculating the probability of simple events

(1) If we have to consider only *one set* of *alternatives*, *each* of the alternatives being of *equal* value, we may express the datum in the single disjunctive proposition, A is a_1, a_2, a_3 $a\eta$. Then the *probability* of *each* of the alternatives may be represented by the fraction $1/n$. This may be explained arithmetically. Suppose that A is either a_1, a_2, a_3, a_4, and that these are all the possible alternatives, and are exclusive and of equal value. Here we have only four alternatives (i.e. n is 4). Then the probability of each of the alternatives ($1/n$) is $\frac{1}{4}$. If there are n alternatives, then the

chances of a particular alternative *not* being realised are $(n-1)/n$. If there are 4 alternatives, then the chances of a particular alternative not being realised are $(4-1)/4$. If an urn contains 3 balls, one black and two white, then the probability of drawing a black ball is $\frac{1}{3}$ and that of drawing a white ball is $2/3$. The probability of a black ball not being drawn is then $(3-1)/3$ *i.e.* $2/3$, and probability of a white ball not being drawn is $(3-2)/3$, *i.e.*, $\frac{1}{3}$.

(2) If *two events* are *independent* and if the probability of one of them is $1/m$ and that of the other $1/n$, then the *probability* of their *happening together* is $1/mn$. If a man meets A once in 5 times and B twice in 5 times, the probability of A and B coming together is $1/5 \times 2/5 = 2/25$. Thus the rule is that if two events are *independent, i.e.* have neither connection nor repugnance, then their *concurrence* may be determined by *multiplying* their separate probabilities. But if A and B meet oftener than twice in 25 times, it is likely that there is a connection between them; if less often, then it is likely that there exists a repugnance between them.

The rule for determining the concurrence of two independent events

(3) In the case of *dependent* events the rule for determining probability is the *same* as in the case of *independent* events. The probability of a coin turning up heads when tossed for the first time is $\frac{1}{2}$, the probability of its doing so when tossed a second time is $\frac{1}{2} \times \frac{1}{2}$, *i.e.*, $\frac{1}{4}$, and when tossed for the third time, $\frac{1}{4} \times \frac{1}{2}$, *i.e.*, $\frac{1}{8}$. To express it symbolically, if the probability of A is $1/m$ and that of B is $1/n$, then the probability of AB is $1/mn$. By such calculations the value of testimony can be determined. Testimony deteriorates when it passes from hand to hand. Suppose that the value of the testimony of A is $\frac{1}{2}$, and that he reports it to B,

The rule for determining the probability of dependent events

the value of whose testimony is also $\frac{1}{2}$, and that B reports to C, the value of whose testimony is also $\frac{1}{2}$; then the resultant value of the testimony of C is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$. "Thus, the probability of the conjoined event is seen to be in this, as in the former case, the product of the probabilities of the separate events."

(4) If two events *cannot concur*, the *probability* of the happening of *either* is the *sum* of the probabilities of *each*.

The rule for determining the probability of either of two events which cannot concur

Suppose the probability of a man dying of fever is $1/5$, and the probability of his dying of cholera is $1/10$, then the probability of the dying either by fever or by cholera is $(1/5 + 1/10) = 3/10$. We have seen that the probability of getting heads when a coin is tossed is $\frac{1}{2}$, and that of getting heads in the second toss, which is contingent on the first, is $\frac{1}{4}$; we now see that the probability of getting heads in either of the two successive throws is $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$.

(5) If a man has taken rice for 1,000 days successively the probability of his taking rice once more is 1000 : 1001.

Determination of probability of recurrent events

Thus we find that with continued uncontradicted experience, the probability of a single repetition of the event rises very high indeed. The value of induction by simple enumeration can be determined by such a calculation of probability.

(6) If A concurs with B and C, and if the probability of the concurrence of A and B is $4/5$, and that of A and C is $4/5$, then the probability of B and C together being the sign of A can be determined by multiplying their improbabilities, *viz.* $(1/5 \times 1/5) = 1/25$, and subtracting this from 1, the result being $(1 - 1/25) = 24/25$.

The value of cumulative evidence

The rule for such a calculation is that if an event coincides with two or more independent events, the probability that these will together be a sign of it is found by multiplying together the fractions representing the improbability that each is a sign of it, and subtracting the product from unity. By this rule we can determine the value of cumulative evidence. If the value of evidence of one witness in a law court is $\frac{3}{4}$, and that of another is $\frac{3}{4}$, then the cumulative value of their joint evidence is $1-(\frac{1}{4}\times\frac{1}{4})=(1-1/16)=15/16$. Here the improbability of the first evidence is $\frac{1}{4}$ and that of the second is also $\frac{1}{4}$; their product therefore is $1/16$; and if we subtract $1/16$ from 1, we get $15/16$.

CHAPTER IX

LAW AND EXPLANATION

Laws

Some preliminary remarks

After our previous discussions, we need hardly repeat that the aim of induction is to discover and prove the laws of Nature, which are the governing principles of all occurrences. The very possibility of science is dependent on the belief that the things of the world are interconnected, and that their mode of operation is governed by laws. Though Aristotle and other ancient philosophers believed in chance happenings, most modern scientists hold that we attribute certain events to chance only because we are ignorant of the laws which govern them. Nothing in the world, says the scientist, is casual, but every-

The attitude of the scientist

thing is causal. Certain phenomena which were once supposed to be due to chance are now explained by causal laws. Superstitions have largely been eliminated with the progress of science. Further, scientists believe not only that every department of Nature is governed by laws, but also that there is interconnection between the different departments. The law of gravity, which once was supposed to be restricted to the phenomena of the earth, has been gradually extended to explain the motions of all the heavenly bodies. Scientists try to refer complex laws to simpler laws in order to discover unity in the universe. So the belief in the uniformity of Nature is required by the scientist. It is a postulate of science, because without this belief the impulse of the scientist to discover laws would die out. The world of the scientist therefore is not a chaos but a cosmos. It is orderly and systematic.

The world is governed by laws

Classification of laws

The laws which the scientist aims at explaining and discovering have been classified into (1) Axioms, (2) Primary laws, and (3) Secondary laws, which are themselves subdivided into (a) Derivative laws, (b) Empirical laws.

Axioms

Axioms are those laws which are supposed to be *self-evident*, *universal* and *necessary*. Some axioms apply to all sciences: such are the laws of thought, *e.g.*, the law of identity, the law of contradiction and the law of excluded middle. These laws, being the universal postulates of knowledge, are assumed to explain both quantitative and

qualitative relations. Mathematical axioms are assumptions which are necessary to explain quantitative relations only, *e.g.*, Things which are equal to the same thing are equal to one another; The part of a thing cannot be equal to or greater than the whole, etc. Furthermore, every science has its special postulates or axioms which hold good within the limits of that science. According to some, the principle of the uniformity of Nature and the principle of causation are inductive generalisations; but we have seen that they should be regarded as postulates of knowledge, though they are not as universal as the fundamental laws of thought. The axioms which are assumed by different sciences are examined and criticised by metaphysics, and it is not the function of logic to pass a verdict upon the assumptions of knowledge.

Primary laws

Primary laws are *less general* than axioms and require *inductive proof*. But they are called primary because their validity has been established in such a way that it is no longer questioned. They are more general than secondary laws, which are explained by being derived from primary laws. Such laws as the law of gravitation, the law of definite proportion in chemistry, the principle of conservation of energy, the principle of heredity in biology, the principle of relativity in psychology, etc., are primary laws. Such laws are believed *not* to be *derivable* from other inductive laws of *greater generality*.

Secondary laws

Secondary laws are concrete and *less general* than primary laws. Some of them, *viz.*, those which are called *derivative*, have been proved by deduction. But the secondary laws which are called *empirical* are *inductions* by *simple*

Derivative and empirical laws

enumeration which have not yet been proved by deduction from primary laws. "An empirical law", says Bain, "is a uniformity supposed to be secondary, that is, resoluble into some more general uniformities, but not yet resolved." Such laws as the laws of the tides or of the weather, the laws of motion, Kepler's laws, the laws of projectiles, the laws of price in economics, etc., are examples of *derivative* laws. The nature of such has been explained in connection with deductive method. Those derivative laws which have been deduced from simple laws are supposed to be of greater value than those which have been deduced from complex laws. Mill says that laws of effects are derivative because they are derived from the laws of their causes. Such generalisations as, Quinine cures ague, All scarlet flowers are without fragrance, Water can be pumped to about 33 ft., above sea level, When different metals are fused together, the alloy is harder than the various elements; some generalisations about weather from the appearance of the sky, some generalisations about local tides, etc., are *empirical* laws.

Examples of derivative laws

Examples of empirical laws

Many laws which were *once empirical* have now *become derivative.* "The occurrence of snow on high mountains was at one time an empirical uniformity, but we can now resolve it into the laws connected with radiant heat passing through the atmosphere." The periodical occurrence of eclipses was an empirical law until general laws of celestial motion had accounted for it. An empirical law is said to have been *explained* when it has been derived or deduced from a pri-

Empirical laws may become derivative

mary law. Though we try to discover derivative and empirical laws for scientific purposes, yet it is undoubted that empirical laws are chiefly of practical value. Bacon rightly observed that in induction we must pass gradually from laws of lower generality to laws of higher generality. Secondary laws are the middle axioms or 'media axiomata' of Bacon, through which we should ascend to primary laws. Conversely we should descend from primary laws to middle axioms, and from these to individual cases. In induction we start with particular facts of experience and from them arrive at empirical laws, which are explained and proved when they are deduced from primary laws.

Bacon's view of scientific method

Some empirical laws are laws of *succession*, *e.g.*, Water quenches thirst, Fire warms, Day follows night or night follows day, Poverty is the cause of many crimes, The taking of nutritious food is the cause of an increase in muscular strength, etc. The first two examples are examples of direct causation, while in each of the last two examples an effect is explained by its remote cause. The example that day follows night or night follows day is an example of mere succession. Some empirical laws are laws of *co-existence*, *e.g.* the law of the co-existence of gravity and inertia, the laws of co-existence of attributes in natural kinds, the law that there is co-existence of deafness with tom-cats having deep blue eyes, the uniformity of co-existence of scarlet flowers with absence of fragrance, etc.

Some empirical laws are laws of succession. Examples

Some empirical laws are laws of co-existence. Examples

Invariable empirical laws

Those secondary laws are said to be *invariable* which apply to all the members of a certain class, *e.g.* All lions are tawny, All arctic animals which require concealment are white, All gold is yellow; while others are said to be *approximate*, the form of these being, Most S's are P; *e.g.*, Most crows are black, Most swans are white, Most arctic animals are white, etc. Secondary laws which hold good within a certain range may only be extended to adjacent cases after great precaution has been taken. But primary laws can generally be extended to new spheres which are beyond their direct range. We have referred to this matter in connection with consilience of induction.

Approximate empirical laws

Secondary laws have limited range of application. Primary laws have generally wide range of application

Inductive sciences are not concerned with those laws which are supposed to be expressions of will

We have remarked in a preceding chapter that in induction we are not concerned with those laws which are supposed to be the expressions of will, whether of an individual, or of the state, or of God. Political laws are uniformities which are often regarded as commands of the sovereign will of the state. These laws, which are backed by force or sanctions, are not the same in all states. Certain religious laws are believed to be commands of God, *i.e.*, expressions of the divine will. But the sciences have nothing to say about these laws. Similarly moral laws are also regarded as expressions of will, and they are described by Kant as categorical imperatives, inasmuch as they are believed to be absolutely binding on all moral agents.

Inductive sciences are directly concerned only with those laws which have been arrived at by observation of and experiment upon facts.

Explanation

Some introductory remarks about explanation

The problem of *explanation* is the same as the problem of induction, and explanation may be regarded as the *goal of induction*. Induction generalises the facts of experience in order to explain them. We have seen that induction can prove its conclusions only when it is aided by deduction. This being the case, explanation requires the aid of deduction as well as of induction. The first attempt to explain occurrences leads to the formation of some hypothesis. We form hypotheses either to explain a particular occurrence or to discover a link between a number of facts. But a hypothesis which is suggested either by simple enumeration or by analogy can explain facts only provisionally before it is proved by deduction and verification. The end of explanation is attained when a suggested hypothesis, accepted to explain facts, eliminates rival hypotheses and is proved to be the only hypothesis capable of explaining the phenomena under investigation.

Hypothesis, induction and explanation

The first attempt to explain facts usually leads us to refer them to some empirical law arrived at by simple enumeration. But an empirical law, before it becomes derivative, can only offer a provisional explanation of phenomena. Induction by simple enumeration, which may lead to the discovery of an empirical law, cannot establish a universal and necessary proposition in the form, If S is M it is P, which induction requires; but it can establish a con-

clusion which can be stated by such a proposition as, S may be P, or S is P. Even analogy, though it goes beyond simple enumeration, inasmuch as it rests upon intension and not upon extension, cannot establish a universal and necessary proposition. The conclusion either of analogy or of simple enumeration cannot therefore provide scientific explanation of phenomena. Induction requires that all the conditions under which phenomena occur should be fully comprehended, that is, that the consequence should be explained by sufficient reason. In explaining the facts of the world we form hypotheses by means of simple enumeration and analogy, and when these hypotheses are proved both by deductive and inductive methods, we arrive at scientific inductions, which alone can sufficiently explain facts.

Explanation elucidated and defined

The object of explanation is to make intelligible what is obscure, and we can explain facts when we can discover rational connection between them. *Assimilation* is necessary for explanation. When facts are assimilated to other facts a step is gained towards explanation. Classification therefore is the first stage in the attempt to explain phenomena. We classify phenomena on the basis of resemblance between them in important and numerous points. Suppose some birds are placed before you and you are called upon to explain them. If after due observation you find that they resemble parrots in many and important points, and come to the conclusion that they belong to the species of birds called parrots, you are supposed to have explained them in a way. But mere assimilation is not sufficient for explanation. To explain we must not only ascertain the points of resemblance, but we must also be sure that they are sufficient to provide the basis for a statement of causal connection. So it is said that the law of an

effect is explained by finding out the law of its cause or the laws of its causes. We know how the deviation of Uranus from its calculated orbit was explained by the discovery of Neptune. Pasteur explained the disease of silk-worms by discovering the corpuscles which were responsible for the disease. Similarly the fall of bodies was explained by Newton when he discovered the law of gravity. Lastly, explanation very often consists in deducing a law from some higher law. Thus an empirical law is explained when it is derived from a primary law. The law of falling bodies is explained by deducing it from the law of gravitation. So Carveth Read says, "Scientific explanation consists in discovering, deducing and assimilating the laws of phenomena." We have already remarked that no one of these processes by itself is sufficient for scientific explanation, which requires the aid of both induction and deduction.

Demonstration a form of explanation

Mathematical explanation is known by the name of *demonstration*. Demonstration is that form of explanation which consists in *proving* a *conclusion* by means of *deduction* from *self-evident* truths or *axioms*.

Distinction between popular and scientific explanations

A distinction between popular or common-sense explanation and scientific explanation is drawn by logicians. *Popular* explanation is not explanation proper, though it satisfies the man in the street. What satisfies an ordinary man does not satisfy a scientist. The plague that broke out in the Greek camp when Troy was besieged by the Greeks, was explained by the wrath of Apollo. But this is not scientific explanation. An ordinary man will explain a disease by certain antecedent circumstances which may not

satisfy a doctor. If anyone asks why cork floats on water, an ordinary man will say that it does so because it is lighter than water, but such an explanation does not satisfy a scientist, who wants to know the cause of an event. Popularly an event is explained by referring it to some agent; such and such a man or thing is said to have brought about such and such a change; but scientists try to explain phenomena by causal laws. So it is said that in *scientific* explanation we pass from 'who' and 'what' to '*why*'. An ordinary man will ask, Who did it? What is it? while a scientist will ask, Why are occurrences so and so and not otherwise? "The explanation by the theory of the law of pressure of gases, and that of the moon's motion by gravitation, are scientific explanations, whilst the illustration of the former by rebounding bodies and the statement of the latter as a movement of continual falling are examples of the popular kind of explanation."

Three recognised forms of explanation

Mill and Bain have distinguished three forms of explanation, *viz.*, *analysis*, *concatenation*, and *subsumption*. (1) *Analysis*—"Explaining a *joint effect*, by assigning the *laws* of the *separate causes*, as in the ordinary deductive operation". We have already explained this matter in connection with deductive method. Such an explanation consists in deducing the law of a complex effect from the laws of its separate causes. The ascent of a balloon is explained by the law of gravity, the law of buoyancy, of gaseous elasticity, the exact weight and elasticity of our atmosphere, and the specific gravity of the mass of the balloon. The fact that thunder follows lightning can be explained by pointing out the remoteness of the observer and the fact that light travels faster than sound. In the chapter

on the Deductive Method other examples have been given of this form of explanation. (2) *Concatenation* or *interpolation* of causes; which consists in discovering an *intermediate* link, or links, between an antecedent and a consequent. When we account for a sensation by referring it to the stimulation of a sense-organ, we must point out how the stimulation of a sense-organ is followed by nervous action, which ultimately is followed by sensation. If we fail to point out the intermediate link between the stimulation of a sense-organ and sensation, our explanation will not be complete. To explain the fact of the communication of ideas between two persons we have to point out the relation between thought and the organic processes through which it is put into language by the one person, and the process by means of which it is received by the other. Similarly, to explain the death of a man after he has been shot through the heart, we must point out the organic changes which are brought about by the bullet entering the heart, and the relation of these changes to death. (3) *Subsumption*, which consists in *deducing* one law or a number of laws *from a law of higher generality*. "Magnetism, common electricity, Voltaic electricity, electro-magnetism etc. are all strung upon the common thread of electrical polarity." "The most splendid example of this operation was when terrestrial gravity and the central force of the solar system were brought together under the general law of gravitation." We have already seen how the law of falling bodies is explained by subsuming it under the general law of gravitation. The law of causation may be explained by deducing it from the law of the uniformity of Nature. We have already mentioned that secondary laws are explained by this process, that is, by deriving them from primary laws.

There are phenomena which *cannot* be *generalised*,

Limits of explanation

that is, from them inductive reasoning is not possible. Such phenomena are those which *cannot* be *assimilated* to other phenomena. We have seen that without discovering resemblance between phenomena we cannot classify them and explain them. So unique phenomena cannot be explained. Therefore the *limits* of *explanation* are the same as the limits of *induction*. Terrestrial gravity and celestial attraction, being similar in nature, can both be explained by referring them to the general principle of gravitation. There are laws according to which human beings generally behave, and these laws can explain the actions of men because those actions exhibit resemblances between themselves. But ultimate laws, such as those of the conservation of energy, the theory of evolution etc., which cannot be assimilated to any other law, cannot be explained. Similarly, elementary sensations and feelings, such as taste, smell, pleasure, pain, anger etc., cannot be assimilated to anything else, and therefore these also do not admit of explanation. So Mill says that the ultimate laws of Nature cannot be less numerous than the ultimate feelings of the human mind, and this is the insurmountable barrier to the further generalisation, and consequently to the explanation, of these laws. Primary qualities of material things, such as extension, resistance, motion, form etc., which are unique, cannot be explained. Such self-evident principles as those of the uniformity of Nature, identity, contradiction, excluded middle etc., cannot be explained, as they are ultimate assumptions of thought. Particular things such as a piece of stone, a particular tree, a particular table or chair, etc. cannot be explained by any law. We can only describe how they have come into being, but we cannot say why they are as they are and not otherwise.

The world as a system

To conclude our discussion of the principles of induction, we may point out that the aim of science is to frame an adequate and consistent conception of the universe as a systematic whole. Mach aptly remarks that scientific thought is 'economical', and makes it possible to grasp mentally in one compendious view a mass of detail which would otherwise be utterly unmanageable. Scientists try to interpret the facts of the universe in accordance with the principles of *simplicity* and *continuity*. Further, Occam's Razor is a maxim to which scientists constantly appeal, that is, they try to explain the phenomena of the world by *as few* principles as *possible*. The theory which requires the fewest ultimate principles to explain the phenomena of the world is supposed to be the best. Joseph says: "In the words 'fewest and simplest' are contained perhaps the most important of the preconceived ideas which we have about the explanation of the facts of Nature". Mill writes that the problem of induction is—"What are the fewest assumptions which being granted, the order of Nature as it exists would be the result? What are the fewest general propositions from which all the uniformities existing in Nature can be deduced"? Mill further admits that induction has to appeal also to the principle of simplicity. Welton thinks that the teleological explanation of the universe is the only explanation in which the mind of man can find rest and satisfaction. But it seems that the sciences can rest content with causal explanation only, and need not appeal to divine purpose to explain the phenomena of the world. It is for the metaphysician to discuss the problem whether the universe displays design or not. The teleological explanation may be ultimately accepted as the most rational view of the universe, but it is not necessary for science to explain the world by such a conception.

The principles of parsimony, simplicity and continuity

CHAPTER X

THE DOCTRINE OF CLASSIFICATION

The Problem of Nomenclature and of Terminology

The problem of *classification* is intimately related to problem of *definition* and of *division*. In the first part of the book the latter problems have been discussed. In studying the problem of classification students will do well to consult the chapters on definition and division. (Part I).

Classification

An explanation of the nature of classification and a comparison between division and classification

We often speak indifferently of division and *classification* in the same sense, but a distinction should be drawn between them. We have already remarked that division involves classification and classification division. Miss Jones therefore is right when she remarks, "Division and classification are the same thing looked at from different points of view", and a table of division is at once a table of classification. But in spite of this there is a distinction between the two. According to Mill the doctrine of division gives place to that of classification when we adopt a material standpoint. In division, as we have found, we cannot altogether ignore facts, yet it is more or less formal and limited in investigating facts. But classification is impossible without putting material consideration

first and foremost. Thus classification, more than division, helps towards the progress of knowledge. In division we pass from unity to multiplicity, in classification from multiplicity to unity, or at least to a system. Joseph distinguishes between division and classification by stating that though division is closely allied to classification and definition, "the difference between division and classification seems to be principally this, that we divide the genus, but classify the particulars belonging to it. In other words, division moves downwards from the more general to the more special, classification upwards from the particulars through the more special to the more general." The problem of classification, according to Mill, is that "classification should be to provide that things shall be thought of in such groups, and those groups in such an order, as will best conduce to the remembrance and to the ascertainment of their laws." Welton explains the nature of classification thus:—"A........development of logical division on the material side leads to the theory of classification. The object of classifying is to so arrange in order the facts with which we are dealing that we can most easily acquire the greatest possible command over them, and can economise statement—and so lighten the task imposed on memory—by being enabled to convey a large amount of information in a few words."

Names and classification

General names such as man, house, tree etc. classify things for us. Predicables and categories, though names of classes, are not the names of natural classes according to Mill. Mill says that there are general names which are not class names, *e.g.*, God, mermaid, ghost, etc., but there are other general names which are really names of classes, such as animal, plant, dog, elephant etc. "There is a classification of things which is inseparable from

the fact of giving them general names'' (Mill). Every connotative name divides things into two classes, *viz.*, those which have this connotation and those which do not have this connotation. We are here concerned not with how names classify things, but with the grouping of things, which are afterwards given general names as a consequence of classification.

The distinction between natural and artificial classification

A distinction is drawn between *natural* classification and *artificial* classification. Natural classification is the grouping of things according to important and numerous points of similarity, while artificial classification is the grouping of things according to some unimportant points of resemblance. Thus it is supposed that nature has herself arranged things in classes, and to group things according to nature's plan is scientific, as in the grouping of animal species according to the degree of perfection of animal life. On the other hand artificial classification is made according to the purpose of the individual concerned, as in grouping words in a dictionary or grouping books in a library catalogue, in alphabetical order. In such a case the members of a group have no inner affinity. The words 'man' and 'mountain' are grouped together in a dictionary, though the things denoted by the terms have no affinity. But the grouping of books in a library in different book-cases according to subject matter, *e.g.*, books on logic, history, physics and so on, is scientific, inasmuch as there is a natural affinity between the members of every group. Similarly Bentham's classification of flowering plants is artificial. He classifies British flora into those whose flowers are compound and those whose flowers are not compound, and sub-divides the former into those with one seed and those with more than one seed,

and so on. Again the classification of plants into monocotyledons and dicotyledons is, according to Mill, artificial.

It may be pointed out that natural classification is objective, because in this case things are classified according to their resemblance or affinity, while artificial classification is subjective, inasmuch as in this case the interest of the individual, that is, the purpose he has in view, determines classification. Thus the classification of plants by a medical man is not the same as the classification of them by a farmer.

Criticism of the distinction between natural and artificial classification

Natural classes were supposed by the schoolmen and others to be fixed and unalterable. But the science of biology has shown that classes cannot be shut up in watertight compartments. Various species and genera can be traced to a common stock and there is a family relation between them. Besides in the course of evolution a particular class may undergo metamorphosis, and change its character altogether. So classification should be effected not according to resemblance but according to family relationship. Modern books, therefore, set out family relationships by means of 'genealogical trees'. Further according to Welton there is no essential difference between artificial and natural classification. Every classification is artificial and is based upon some purpose, yet it aims at being natural. We classify according to some idea. He distinguishes however between classification for some special purpose, analogous to artificial classification, and classification for a general purpose, similar to natural classification. Besides, we may remark that artificial classification is not useless, but has proved very valuable in many cases. Instead of speaking of natural and artificial classification we should rather distinguish between *scientific* and *popular* classification. Botanists and Zoologists

have attempted to give us scientific classifications of plants and animals, and their attempts have proved very valuable for the progress of knowledge. We should also remark that classification should not be made once for all. Old classifications must give way to new ones with the advance of knowledge. The old classification of plants into trees, shrubs and herbs has been found to be unscientific.

The rules of classification

In the light of what has been said above we may provide some rules of classification. The rules of division given previously hold good for classification as well. The classes should be exclusive, and the sub-classes together should be equal in extent to the genus divided. In classifying a single principle should be observed, etc. Besides these, we may note certain special rules of classification, *viz.*—(1) "The classification should be appropriate to the purpose in hand" (Welton). (2) The higher the group the more important should be the attributes by which it is constituted. (3) The classification should be graduated, so that the groups with most affinity with each other may be nearest together, and so that the distance of one group from another may be an indication of the degree of their dissimilarity. (4) Groups should be so constituted as to differ from each other by a multitude of attributes. These rules are difficult to satisfy. How are we to find out the most important attributes? Our knowledge is not perfect. To arrange classes in a gradual order must depend upon knowledge of the affinity between different classes. It is very difficult to find out this affinity.

According to Whewell, natural classes are given by type and not by well defined characters. Species are not clearly marked off and one runs into another. Natural

Classification by type and some criticism of it

history provides us with classification according to type. A type is an example of a class which embodies in a prominent degree the leading and important characteristics of the class. Thus in classifying monkeys according to type we may take one typical example of the class, which has the leading characteristics of the class. Natural classification is thus not determined from without but is determined by a central point within, that is, by internal affinity and not by external marks. Now, according to Jevons, the type is an individual and no other individual is like it. If a type is found out by the selection of a few important qualities, this can only be the result of a knowledge of classification, that is, by a knowledge of the connotation of the class name. Thus classification by type is not possible without a knowledge of the general attributes of a class. Thus an independent classification by type cannot be defended logically. But classification by type is not, therefore, necessarily useless.

Classification by series

Comte was the first to recognise classification according to *series*. Classes are marked by mutual affinity and if they are serially arranged, it becomes easy to find out very general laws. How to arrange the classes side by side, not how to arrange individuals, is the problem of classification by series. One species often passes into another and this again into a third and so on. Thus to arrange groups according to nearness of relation is serial arrangement. An ellipse passes into a circle when its diameters become equal. It passes into a straight line when the conjugate diameter becomes nil. An ellipse is thus intermediate between a circle and a straight line. It rather resembles an arrangement of concentric circles in a globe. Classification in zoology

is according to the degree of perfection of animal life. A species is perfect, not when it is between two extremes but when it passes to some other species. Thus we do not have a series but a graduated order in natural classification. Man is not intermediate between animal and dog, but in relation to animal both man and dog have the same position.

The uses of classification

Scientific classification, aided by scientific nomenclature, contributes largely to the advancement of knowledge. The things of the world are innumerable and varied. To group them under classes is a great aid to memory. Without classification they would become unmanageable. Classification helps us to find out general laws, without which knowledge cannot progress. Again if classes are arranged in graduated order according to affinity and nearness we can compare conveniently and draw inferences by analogy. Thus zoology and botany give us genealogical trees.

Nomenclature and Terminology

Words in general use change their meanings; scientists when they use them ought clearly to define their meaning or else invent technical terms

Names invented by science are well defined and have their meanings fixed; *e.g.*, the terms point, triangle etc. in geometry. But the meanings of names popularly used vary from time to time according to usage and custom. Thus we find in the dictionary that a word has different shades of meaning. The word 'pagan' originally meant a villager, but now it means a heathen. Fidelity meant faithfulness to the oath of allegiance, but now it means any kind of faithfulness. These examples show how words undergo a change in their

meaning. Words change their meaning either by *generalisation* or by *specialisation.* When words are generalised their connotation diminishes. Oil which originally meant olive oil, now means any kind of oil. This is an example to show how a word changes its meaning by generalisation. Psychologists and sociologists often generalise words. When words change their meaning by specialisation, their connotation increases. Logicians should accept words in general use, but should define them clearly and precisely for the fulfilment of their purpose.

The problem of nomenclature explained

Nomenclature is a system of names for classes. According to Mill, "A nomenclature may be defined as the collection of the names of all the kinds with which any branch of knowledge is conversant, or more properly, of all the lowest kinds or infimae species—those which may be sub-divided indeed, but not into kinds, and which generally accord with what in natural history are termed simply species." No classification can remain fixed without a corresponding nomenclature, and every good nomenclature involves a good system of classification. Whewell says, "System and nomenclature are each essential to the other." If classes lack nomenclature, the progress of thought becomes impossible. Names of classes have not only denotation, but they have also connotation conventionally fixed. Though artificial classification may have nomenclature, such nomenclature is not scientific, since the same thing may be artificially grouped under different classes. Botany, zoology and chemistry give us excellent systems of names or nomenclatures for classes. In botany higher classes have been given names such as Dicotyledon, Rosa, Geranium etc. "The species is marked by adding a distinctive attribute to the name of the genus, as viola odorata, or

chis maculata etc." (Welton and Monahan). These distinctive attributes do not stand for differentiæ. Sometimes a class is named after some individual, *e.g.*, Rosa Wilsoni; sometimes after some country, *e.g.*, Anemone Japonica; sometimes from some peculiarity of the plant, as Geranium Sanguineum; some names are fanciful, *e.g.*, Bauhimia. We have from the time of Linnaeus the names of higher classes of plants. Chemistry gives us another method of naming classes, based upon the oxygen theory. The principle here adopted as the basis of scientific nomenclature is founded on a modification of the relation of elements. We have thus "sulphuric and sulphurous acids, sulphates and sulphites of bases, and sulphurets of metals; and in like manner, phosphoric and phosphorous acids, phosphates, phosphites, phosphurets" (Welton and Monahan). Such naming at once implies the place of a thing in a system. The three oxides of iron are protoxide, the black oxide and the peroxide.

The problem of Terminology plained

We should not only have scientific names for classes but also scientific terms to be able adequately to describe individual things. Such a system of scientific terms necessary for the description of individual things is called *Terminology*. Thus while nomenclature is a system of names for classes, terminology is a system of names for parts or qualities of individual things. Botany provides us with examples of such a terminology. Here also we are indebted to Linnaeus. Thus the parts of flowers have been distinguished as calyx, corolla, stamens and pistils. Names of the parts of plants are pistil, stamen, calyx, frond, and names of properties are bipartite, silicate, pinnate. When we use current names, we should fix their meaning conventionally, and technical names should be well defined and taught through the knowledge of objects.

CHAPTER XI

METHOD

General Nature of Method

We have so far discussed the two types of inference, *viz.*, Deduction and Induction, and have also pointed out that every science is indebted to logic inasmuch as logic acquaints us with the conditions of valid thinking. But for the attainment and exposition of truth mere correctness of inference is not enough. Not only should we know how a correct conclusion may be drawn from certain premises, but our inferences should be properly and *methodically arranged*. *Method* is defined as "the correct arrangement of thoughts either for the discovery or for the exposition of truth". A science which employs a faulty method can never establish conclusions which are true and exact. Science aims at accurate and exact knowledge, but if its method is itself unscientific, it can never attain truth. Every science requires consistency of thought, and if its reasonings are not consistent with one another it must fail to attain truth. Correct method is necessary also for exposition, that is, for the deduction of a conclusion from a given truth. Some sciences, such as mathematics, physics and chemistry, can rigorously employ scientific method, while others, such as the social and historical sciences, cannot so rigorously employ it. The logic of method arises from the study of the particular methods employed in different sciences. So Welton and Monahan

Method defined and its nature explained

say, "The establishment of the logic of method is itself a scientific work which uses as its material the *special methods* of the *various sciences* which deal directly with the facts of the world."

"The character of the *method* to be adopted is partly *determined* by the *subject-matter* dealt with, and partly by the *purpose* we have in view." We know that in some sciences we begin with general propositions, and our object is to apply them to particular cases; while in others we begin with particular instances and try to discover what general laws those particulars exemplify. The former sciences are deductive, the latter inductive. In the *deductive* sciences we follow the *synthetic* method, whereas in *inductive* sciences we make use of the *analytic* method. In deductive sciences we descend from the more general to the less general, while in inductive sciences we ascend from the particular to the general. A science which has to deal with concrete facts must begin with the experimental or analytic method, but as it gradually progresses it becomes deductive and follows the synthetic method more and more. Thus physics, which was originally an experimental science, has in the course of its evolution become more and more deductive and abstract, and pure physics is analogous to the science of mathematics. In pure physics exact quantitative determination is possible. Thus an empirical science, when it admits of exact quantitative measurement, ceases to be experimental and becomes pre-eminently a deductive science like mathematics. Thus pure physics, astronomy and geometry are synthetic sciences, while chemistry, geology and medicine are experimental or analytic. The *analytic* method is also called the method of

The two main methods, Synthesis and Analysis, which correspond to deductive and inductive reasoning respectively

discovery, while the synthetic method is called the method of *exposition*.

But though this distinction is plausible, yet sciences really require the help of *both* methods, and knowledge advances by a combination of analysis and synthesis. The method of exposition is used to explain matters to one who can grasp general truths. In educating children, the method of analysis is followed first so as to acquaint them with concrete facts; but when they grow up and can grasp abstract truths, the method of synthesis is followed for exposition.

The meaning of Methodology

Every science has its peculiar problems, and *rules of method* are followed to deal with these difficulties. "The *consideration* of such *rules*, as distinct from the use of them, is *Methodology*."

Descartes' general rules of method stated and explained

Descartes has given us four general rules of method which have been generally accepted. These are: (1) "Never to accept anything as true which we do not clearly know to be so." This rule forbids us to start an enquiry with propositions about the truth of which we are in doubt.

(2) "To divide each of the difficulties under examination into as many parts as possible and as may be necessary for its adequate solution." This rule requires that in order to solve a complex problem we must analyse it thoroughly, and study with care and attention every subordinate problem which comes under it. We should carry on our analysis adequately in conformity with the purpose in view. A chemist, in order to solve his problem, has to analyse a complex phenomenon into its component parts. Aristotle fol-

lowed this method. To explain the nature of the state, he began with the study of families and villages, which are the constituents of a state.

(3) "To conduct our thoughts in such order that, by commencing with objects the simplest and easiest to know, we may ascend by little and little, and, as it were, step by step, to the more complex, assigning in thought a certain order even to those objects which in their own nature do not stand in a relation of antecedence and sequence." This rule requires us to start in our investigation with simple propositions and to pass from them to complex ones in an orderly and gradual manner. But what is simple to a great scientist may not be simple to an ordinary man. To Newton the principle of gravitation was simpler than the fall of an apple. But to the man in the street the fall of an apple is simpler than the principle of gravitation. To some minds there is nothing so simple as the most abstract laws, while others regard concrete facts as simpler. So we may either begin with the abstract and pass from it to the concrete, or begin with the concrete and pass from it to the abstract, following either the synthetic or the analytic method according to our need. But whether we start with the abstract or with the concrete, we must always proceed step by step in our investigation if we are to arrive at a correct conclusion. If we do so, we can discover order in the phenomenal world; and science aims at discovering order.

Descartes' fourth rule is : (4) "In every case to make enumerations so complete and reviews so general, that we may be assured nothing is omitted." This rule requires that observation should be thorough and complete. But we know that in science careful analysis of facts is more important than observation of a large number of instances.

To these four rules of Descartes another may be added, which is: (5) "To conceive clearly the end to be attained by the enquiry or argument." This rule is very important. It lays down that success in scientific enquiry largely depends upon the clear perception of the end in view, and also upon being guided at every step of our enquiry by the purpose in hand.

These rules are general, and apply both to analysis and synthesis, but because of their very generality they cannot give us practical guidance at every step of our enquiry. However if we bear them in mind and are guided in our enquiry by the purpose in hand, we may expect success. "The essence of the rules may be summed up in the directions to make sure of our *starting point*, to know the end we wish to attain, and to go from the starting point to that end by *orderly* and *consecutive steps*, each of which is seen in its true relation to all the rest of the enquiry."

The essence of the rules

CHAPTER XII

ANALYSIS AND SYNTHESIS

Analytic Method

Analytical method explained

Analytic method is really the method of *inductive* enquiry. In discovering laws, scientists naturally start with the analysis of the concrete. Here the 'simple' with which enquiry begins is particular facts of experience. Welton remarks that analysis is not confined to single steps of inductive inference, but often involves long and complex trains of reasoning. Successful analysis depends upon sagacity and judgment rather than upon rules.

Successful analysis requires previous knowledge, scientific training and accurate judgement

Knowledge, scientific training and accurate judgment are necessary for successful analysis. Instruments are often used for analysing facts, and only scientists can use them properly, for they embody much knowledge.

Scientific analysis is never haphazard but always guided by some purpose. A complex phenomenon is analysed into its constituent elements in order to determine its proper character. Analysis acquaints us with many truths which may conduct us to the knowledge of what we seek. Analysis of facts is a very difficult task. Owing to false analysis men often frame hypotheses which are ultimately rejected. The formation of a good hypothesis

undoubtedly depends upon the correct analysis of facts. It is only by means of successful analysis that we can discover causal laws.

Analytic method employed in exposition and in discovery

The analytic method of *exposition* differs from the analytic method of *discovery*. The teacher who is already acquainted with a truth and with the processes by means of which it came to be known, can select facts conveniently for the advantage of his pupil, but an actual discoverer cannot so easily discriminate between relevant and irrelevant instances. The nature of the analytic method, which is the same as the inductive method, has been fully considered in the course of our discussion of the principles of induction.

Synthetic Method

Just as induction and deduction are not antagonistic, *analysis* and *synthesis* also are *not antagonistic*. They may be contrasted no doubt, but the truth is that one leads to the other. We have already remarked that analysis is generally resorted to for the purpose of discovering truth, synthesis generally for the purpose of expounding it. We have also pointed out that a science which was originally mainly analytic may gradually in the course of its evolution become mainly synthetic.

Analysis and synthesis compared

One of the most important functions of synthesis is to correct the deficiencies of analysis. We know that a law discovered by the analytic method is ultimately proved by the method of synthesis. So to argue from principles in order to establish a synthesis between these principles and particular instances of them, is not less useful than dis-

covery by means of the analysis of facts. "Analysis and synthesis increase understanding, that is, they are methods of explanation." As the scientist proceeds with his investigations he synthesises his results with others in his own subject, and, where possible, with those of other subjects. The subject-matter of a science largely determines whether its method should be mainly analytic or synthetic.

The nature of synthetic method

The 'simple' with which synthesis starts is found in general propositions accepted as true. *Synthetic* method, being the method of deductive enquiry, consists in *applying general principles* to *particular cases*. The general principle with which synthesis starts must be at the outset clearly comprehended. Reasoning required by the synthetic method should also be demonstrative.

The rules of synthetic method

Eight special rules of synthetic method are formulated:

"Two Rules touching Definitions:

Rule 1. Not to leave any terms at all obscure or equivocal, without defining them.

Rule 2. To employ in definitions only terms perfectly known, or already explained.

Two Rules for Axioms:

Rule 3. To demand as axioms only things perfectly evident.

Rule 4. To receive as evident that which requires only slight attention for the recognition of its truth.

Two Rules for Demonstration:

Rule 5. To prove all propositions which are at all obscure by employing in their proof only the definitions which have preceded, or axioms which have been granted, or propositions which have been already demonstrated.

Rule 6. Always to avoid the equivocation of terms, by substituting mentally the definitions which restrict and explain their meaning.

Two Rules for Method:

Rule 7. To treat of things, as far as possible, in their natural order, by commencing with the most general and simple, and explaining everything which belongs to the nature of the genus before passing to its particular species.

Rule 8. To divide, as far as possible, every genus into all its species, every whole into all its parts, and every difficulty into all its cases."

Some explanation of the rules

The first two rules, being the rules of *definition* which have been discussed in Book I, do not require further explanation. The third and the fourth rules state the nature of *axioms*. Axioms are self-evident propositions, and only those propositions are self-evident the truth of which is not derived from any other truth, that is, the truth of axioms is perceived at once without much attention. The fifth rule requires that to prove a proposition which is obscure we must take the help of propositions which are clear, such as definitions, axioms, or propositions which have been proved beyond doubt. Violation of this rule leads to the fallacy of petitio principii. The sixth rule requires that in logic every term should be clearly defined so as to avoid ambiguity or equivocation. We must not use the same term in different senses in the course of the same argument. The seventh rule requires that in synthesis we must follow the natural order, which consists in passing from the more general to the less general, that is, from the whole to its parts, in a gradual manner. We must explain the nature of the genus before passing from it to particular species. The whole with which we start must be

clearly explained before we pass to the parts. The eighth rule tells us what should be the nature of logical division. The parts or sub-classes into which the whole or the class is divided should be together equal in extent to that which is divided.

CHAPTER XIII

METHODS OF THE SPECIAL SCIENCES

Introductory remarks

The sciences may be divided into three special groups, *viz.*, *mathematical*, *physical* and *historical*, according to differences in their subject-matter. Though some sciences are pre-eminently synthetic and others pre-eminently analytic, yet all alike require in some degree the help of both analysis and synthesis.

Methodology of the mathematical sciences

"In mathematics, logically viewed, there is afforded the most consummate exemplification of a formal deductive science." Mathematics requires definitions, axioms and demonstration. It draws conclusions from self-evident principles. *Mathematics* therefore is a *deductive* science and requires *synthetic* method. Both geometry and the science of number, which are the two main branches of mathematics, require deductive operations. The concepts used by mathematics are highly abstract. They "have been discerned by the analytic power of the mind, that it may deal with precision with those spatial and quantitative characters given in experience". The process

of passing from its original empirical character to its present abstract character has been in two directions: "1. the analysis of its concepts, 2. the synthetic building of constructions upon selected sets of the analysed concepts." Both in theoretical and in applied mathematics the method is essentially synthetic, *viz.* the derivation of conclusions from principles. Mathematics uses definite, clear-cut and accurate symbols, ordinary language being imperfect and cumbrous.

Methodology of physical sciences

Those of the *physical* sciences that deal with inorganic Nature are also abstract, though less so than the mathematical sciences. The physical sciences are sub-divided into the *mechanical* and the *chemical*. "The aim of the physical sciences is to understand the inorganic world by bringing its phenomena under far-reaching generalisations." Though these sciences were originally mainly analytic, the *synthetic* element in them has gradually become more prominent.

Methodology of natural sciences

The theory of evolution has given a new turn to those sciences which deal with living organisms. In these sciences analysis is insufficient. To explain the evolution of living beings, analysis must be aided by mental construction. The Darwinian principle of natural selection and the principle of heredity have not been finally accepted. There is still a controversy whether acquired characteristics are inherited or not. Further, there is now an increasing belief that life has developed from non-vital phenomena. For the perfecting of these natural sciences greater study of facts and more searching analysis will be necessary. Biology is a science still in the making.

Natural science is intermediate between the physical and the *historical* sciences. History studies the thoughts and actions of men. In studying history we have to study human purpose, which is a new element not studied by other sciences. "As the matter of the physical sciences is more concrete than that of mathematics, and the matter of the natural sciences more concrete than that of the physical sciences, so the matter of history is more concrete than that of the natural sciences." History cannot make any use of experiment, and but little of direct observation. Therefore many historical incidents have to be reconstructed from the data provided by testimony, and here the chances of error are great. In the mental construction which is required by history, objective reference is as essential as in the physical and natural sciences. The method which is employed in history to ascertain and verify facts is somewhat peculiar. Classification is the preliminary step necessary to systematise history. The weighing of testimony in history is a very difficult task. "Records must be examined critically, and guesses or hypotheses formed as to what the facts which they reveal or disguise really were; and these hypotheses tested by relating them to other facts similarly suggested or more surely witnessed by language, laws, buildings, artistic products." To explain historical occurrences we have to study the motives, intentions and actions of men. Causal explanation should be offered wherever possible; when this is not possible owing to scarcity of facts, mental construction should be resorted to. "The personal factor in histori-

Method in the historical sciences. History has to study human motives, which are not studied by other sciences. The matter of history is more concrete than the matter of any other science

Mental construction plays an important part in history

cal construction is a greater disturbing element than the personal equation in the investigations of physical and natural science.'' So history requires both the analytic and the synthetic method. It tries to find out the causes of effects by deductive reasoning, but it has also to synthesise the results by mental construction.

History requires both analytic and synthetic method

It is very important that in studying history we should have special acquaintance with the method to be followed. The data of history should be properly obtained and synthetically arranged. These are "records, monuments, oral traditions, documents, from which the historian has to construct his primary facts''. *Every science* to which the concept of *evolution* is applicable is *historical*, because it cannot omit time-direction. Sciences that can omit time-direction are exact and those that cannot are inexact. "An historical fact is localised at a given time in a given country; it must be reconstructed from the documents with the aid of the historian's imagination and in the light of his knowledge as to what is likely to have happened.'' When historical facts have to be elicited from records and documents, we have to employ deductive reasoning. Analogy has also to be resorted to in history to provide descriptive concepts, such as the "dawn'' of the middle ages, or the "decline and fall'' of a state, etc. The historian's text is often a copy of a copy of a copy and so on, and not the original manuscript. In course of copying, modifications of the original are often made.

The data of history

Deductive reasoning in history

Social sciences also historical

In the *social* sciences, such as anthropology, ethnology, economics, sociology etc., the primary data are extremely complex. In sociology exploration takes the place of experiment. In the social sciences hypotheses have to be discovered. They must be comprehensive and consistent with facts already known.

Comparative method and evolutionary method

Comparative method is applied in biology and the social sciences. The comparative method is not different from the *evolutionary* method. Wherever continuity and development have to be traced, the comparative method can be used with advantage. "The comparative method so regarded is a method of suggesting an explanatory hypothesis, which must be developed and tested in accordance with the usual logical conditions." In comparative method the conditions investigated are varied as much as possible in order to eliminate what is irrelevant. Mere making of comparisons is not, however, comparative method. The comparative method is largely used in studying religion, jurisprudence and other social institutions.

CHAPTER XIV

SHORT SURVEY OF THE DEVELOPMENT OF INDUCTION

The science of induction is of recent growth. We have seen that induction is the analysis of the process of thought by means of which we acquire a systematic knowledge of the universe. We have also seen that Aristotle's induction is nothing but a syllogism in the third figure. He considered only what is usually known as perfect induction, and did not discuss the problems of induction proper. According to Aristotle, universals exist in particulars, but his particulars are not individual things but species. Aristotle's inductive syllogism is possible if complete enumeration of instances is possible. But complete enumeration of individuals is impossible in many cases. Aristotle however believed that exhaustive summation of the species which compose a genus is quite feasible. He also recognised analogical argument, *e.g.*, statesmen should not be chosen by lot because athletes are not so selected. According to Aristotle it is on a rational connection of concepts that the cogency of analogy and induction rests.

Aristotle

The Scholastics however made enumeration the basis of induction, and hence arose the distinction between perfect and imperfect induction. This distinction continues even now. According to Hamilton, the formal axiom "What belongs, or does not belong, to all the constituent parts, belongs, or does not belong, to the constituted whole", is the basis of inductive inference. This dictum is the converse of the

Scholastics

'dictum de omni et nullo'. Aristotle rightly points out that if induction (that is, perfect induction) is to be valid, the minor premise must be simply convertible. The Scholastics however failed to see that logic is the analysis of the process of thought by which real knowledge is organised.

Bacon

Francis Bacon is usually regarded as the father of induction. He could not completely shake off Scholasticism. According to him, the acquisition of all knowledge is a process of interpreting Nature. Bacon says that from the existence of particular instances we must pass gradually to the highest generalisations or ultimate axioms, that is, we should ascend from particular instances to middle axioms and from them to ultimate axioms. A man should be free from all 'idols' or prejudices if he is to be a true investigator of scientific problems. The fundamental methods of Bacon are analysis and elimination. Physical forms, according to him, should be found in the spatial and temporal relations of bodies. The object of induction is to attain knowledge of causes, *i.e.*, of forms. So Bacon condemned induction by simple enumeration as puerile and precarious, since it is overthrown by any contradictory experience. Bacon's three conditions of scientific investigation are: (1) natural phenomena should be experimented upon and a record of the result obtained should be prepared; (2) the facts so obtained should be arranged in an orderly manner so as to observe instances in which a property is present, in which it is present in varying degrees, and in which it is absent; (3) he also prescribed the specially careful observation of 'prerogative instances', that is, instances of great value. He held that by careful observation, and by varying circumstances, forms and laws can be discovered. His method of inductive inference is however syllogistic. He deduces his conclusion from a disjunctive major premise, *e.g.*, X is either A or B or C or D etc., but X is not either B or C or D etc., therefore X is A. This shows that Bacon, in spite of his insistence upon obser-

vation and experiment, was a Scholastic. His method is not the same as the modern scientific method. Bacon and many other empiricists have propounded a mechanical view of knowledge. Bacon's aim was throughout practical or utilitarian. He did not understand the value of the pursuit of knowledge for its own sake, which is the really scientific attitude.

Newton

Newton dealt with scientific method or the method of analysis, though he is mainly famous as a mathematician and physicist. In every science the method of analysis should precede the method of composition. Analysis consists in observation and experiment and in drawing general conclusions from facts observed. By means of it, says Newton, we pass from effects to causes and from less general truths to more general truths. Synthesis, according to him, is deductive reasoning. By means of synthesis general truths are verified. Both analysis and synthesis are necessary for scientific knowledge. When Newton says that hypotheses have no place in experimental philosophy, he means by hypotheses wild generalisations. He did not however condemn scientific hypotheses, for the formation of which he laid down four rules, as we have previously seen. It is indeed Newton and not Francis Bacon who framed the method used by modern natural sciences.

Mill

Mill as an empiricist made sensations or feelings the basis of all knowledge. Empiricists cannot explain why and how knowledge is objective. We have already discussed Mill's view of induction. According to him, in induction we pass from the known to the unknown, from the observed to the unobserved. Mill says, "Induction......is that operation of the mind, by which we infer that what we know to be true in a particular case or cases, will be true in all cases which resemble the former in certain assignable respects." His

induction is not the perfect induction of the Scholastics. "The essence, then, of induction is, with Mill, a transcending of the impressions given in experience." We have seen that the uniformity of Nature, according to Mill, is the ultimate major premise of all induction. He holds that a single observation will give us a valid induction, if the analysis of the conditions is accurate and complete. He does not however ignore the importance of the observation of a large number of instances. Mill however regards the uniformity of Nature as itself an induction, but this view is paradoxical. If causation itself, which is the ground of induction, is of doubtful validity, then induction cannot establish universal and necessary propositions. So Sigwart says: "Taking away with one hand what he gives with the other, Mill shows in the uncertainty of his views the helplessness of pure empiricism, the impossibilty of erecting an edifice of universal propositions on the sand-heap of shifting and isolated facts, or more accurately, sensations; the endeavour to extract any necessity from a mere sum of facts must be fruitless."

Whewell

"The controversy between Mill and Whewell as to the nature of induction was partially due to different conceptions of its scope." Whewell held that induction is the science of discovery, while Mill regarded it as the science of proof. Mill regards perception as a passive process of the mind, while Whewell rightly regards it as both passive and active. Whewell is throughout consistent, but Mill is not so. When Whewell regarded induction as colligation of facts, he insisted upon the constitutive work of thought. "At any..........step of Induction........the inductive proposition is a Theory with regard to the Facts which it includes, while it is to be looked upon as a Fact with respect to the higher generalisations in which it is included." "Facts are phenomena apprehended by the aid of conceptions and mental acts, as theories also are." "We commonly call our observations Facts, when we

apply, without effort or consciousness, conceptions perfectly familiar to us: while we speak of Theories, when we have previously contemplated the Facts and the connecting conception separately, and have made the connection by a conscious mental act." In every act of induction a conception is superimposed upon facts, and this is what is meant by colligation of facts. "All science may rightly be described as progressive colligation of facts" (Green). The conceptions by which facts are colligated are hypotheses. These conceptions are not external to facts. When a hypothesis can explain facts belonging to different departments of study, it becomes highly probable. This function of hypothesis Whewell describes as consilience of induction. "The consiliences of our inductions give rise to a constant convergence of our theory towards Simplicity and Unity." There may be a genealogical table of all inductions from the lowest to the highest. Whewell's view of scientific method is the same as that of Newton.

Jevons

Jevons also is an empiricist. Induction according to Jevons is essentially enumerative. He bases the whole process of induction upon enumeration and the mathematical doctrine of probability. He does not base induction upon the conception of Nature as a systematic unity. According to him all inductive reasoning is the inverse application of deductive reasoning. He thinks that deductive verification of hypothesis is the essential problem of induction. In this respect Jevons is at one with Whewell. He lays down conditions of induction, which are—"1. framing of hypothesis, 2. deduction of consequences from it, 3. observing whether the consequences agree with the particular facts under consideration." (See Welton.)

CHAPTER XV

INDUCTIVE FALLACIES

In Part I, Book III, Chapter VII we explained the nature of fallacy and its place in logic. There we gave a list of various classifications of fallacies given by logicians, and adopted a particular classification which appeared convenient to us for the discussion of the problem. We considered formal fallacies under the following three main heads, *viz.* 1. Non-inferential logical fallacies, 2. Inferential formal fallacies, and 3. Semi-logical fallacies. In this chapter we shall discuss the remaining fallacies under two main heads, *viz.* 1. Fallacies incident to Induction, and 2. Non-logical or Material Fallacies.

Fallacies Incident to Induction

Without correct observation correct inductive inference is not possible. But men do not always observe correctly, and consequently fallacies incident to observation occur. Mis-observation, says Mill, may be either negative (non-observation) or positive (mal-observation). "It is *non-observation* when all the error consists in overlooking or neglecting facts or particulars which ought to have been observed. It is *mal-observation* when something is not simply unseen, but seen wrong; when the fact or phenomenon, instead of being recognised for what it is in reality, is mistaken for something else." Hasty

Non-observation and mal-observation are the two forms of fallacy incident to observation

generalisation is often due to non-observation. We sometimes observe only those instances which are favourable to the establishment of a certain conclusion, and neglect those which may go against it. In induction by simple enumeration we ignore negative instances, and such generalisation involves the fallacy of non-observation. If we say that a soothsayer is a prophet as a result of taking note only of those of his predictions which proved true and ignoring those which proved false, we commit the fallacy of non-observation. Superstitious persons such as believers in astrology, in dreams, in omens etc., very often commit the fallacy of non-observation. If in arguing that dreams have a prophetic character account is taken only of those instances that have been borne out by facts, while those that have proved false are ignored, then the fallacy of non-observation is committed. If a phenomenon is very small, it may escape observation. This was the reason why argon, a constituent of the atmosphere, was not known for a long time, though it was found that nitrogen obtained from the atmosphere was heavier than nitrogen obtained from other sources. Oracles, fortune-tellers, and quack doctors are guilty of non-observation because they wilfully ignore instances which are not favourable to them. In induction both positive and negative instances have to be carefully observed. But it is a natural tendency of men to observe only affirmative instances and neglect negative ones. Owing to this men are slaves of their preconceived notions, which lead them astray and make them commit the fallacy of non-observation. This is the reason why historians often give different accounts of the same event. It was supposed up to the time of Adam Smith that the purchase of foreign goods was inimical to a country's inter-

No-observation is due to bias

Non-observation due to the neglect of observing both positive and negative instances

est. Those who held this view ignored the fact that with the increase of imports, exports may also concomitantly increase. "In every examination of concrete reality a residue of unanalysed elements remains, and this opens out a possibility of leaving out of consideration some element which is an essential part of the phenomenon we are investigating." In observation we should eliminate only irrelevant circumstances and not those conditions which are operative to produce the effect with which we are concerned. Digby's sympathetic powder is an interesting example of this kind of non-observation. When a wound was inflicted on a person with a weapon, Digby would ask the patient to keep the wound clean and cool, and he would regularly dress the weapon which inflicted the wound with the salve of the powder. In this way he was supposed to heal the wound. But the real cause of healing was not the dressing of the weapon, but the keeping of the wound clean and cool and uncovered. When it is said that prodigality encourages industry and thrift discourages it, one very important fact is ignored, *viz.*, that the money saved by the thrifty man passes from hand to hand through the bank, and thus gives permanent employment to many.

Further consideration of non-observation

In *mal-observation* sense impressions are wrongly interpreted. In the case of mal-observation it is not that something is not seen, but that something is seen wrong. When a piece of rope is mistaken for a snake, or a shell shining in the sun for silver, the fallacy of mal-observation is committed. In mal-observation the fallacy is one of wrong inference. It is not wrong to mingle inference with perception, but it is wrong not to see that there has been

The fallacy of mal-observation

such a mingling. The geocentric theory was discarded because it was found to contradict sense experience. This fallacy is generally committed by untrained men. Welton and Monahan write, "The rustic who takes a tomb-stone brightened by the rays of the moon for a ghost, or who interprets a donkey's bray as the voice of a departed ancestor, falls into the fallacy we are now considering." It is due to mal-observation that men think that objects immersed in water are magnified, or suppose that magnets exert an irresistible force, or that a body twice as heavy as another falls twice as fast. To avoid mal-observation we must be equipped with as much knowledge of the thing observed as possible.

False generalisation

Men often generalise without knowing the ground upon which the generalisation is based. Thus when men resolve all things into unity, as the Greeks did when they supposed that everything in the world is a modification of one fundamental principle such as earth, air, fire, or water, they commit the fallacy of *false generalisation.* Materialists try to reduce everything to material atoms, while idealists try to resolve everything into consciousness. The tendency to generalise without adequate evidence may lead to false generalisation.

Induction by simple enumeration leads to the fallacy of false generalisation

The second mode of false generalisation is exemplified in induction by simple enumeration. Ordinary men, as we have seen, by observing positive instances only, hastily generalise and thus commit the fallacy of false generalisation. Bacon strongly condemned such unscientific generalisation, of which the following are examples. It is falsely argued that since black men are at present less civilized

than white men, they will always be inferior to the latter in civilization. Similarly if it be argued that since the first three sons of a particular man have been successful in life, all his sons will be equally successful, there is the fallacy of false generalisation. Mill writes that as a result of false generalisation men in the time of Aristotle believed that society could not prosper without slavery. It is also a false generalisation that philosophers are unfit for business. All empirical laws however are not equally unscientific. The law of Malthus that with the growth of population poverty proportionately increases is highly probable.

The fallacy known as post hoc ergo propter hoc

Some fallacies are committed owing to the *false analysis of causal connection*. The most obnoxious form of this fallacy is what is known as *post hoc ergo propter hoc* (after this, therefore caused by this). It may be argued that the cause of the prosperity of a country is its protective policy, because soon after this policy was adopted the country became prosperous. Such an argument may be false, because in such a case the immediate antecedent of the phenomenon is supposed to be its cause, though it may be only a condition among other conditions. Similarly it may be wrongly argued that since a man fell ill soon after his return from a journey, the journey was the cause of his illness. In this connection Mill writes: "All the doctrines which ascribe absolute goodness to particular forms of government, particular social arrangements, and even to particular modes of education, without reference to the state of civilisation and the various distinguishing characters of the society for which they are intended, are open to the same objection—that of assuming one class of influencing circumstances to be the permanent rulers

of phenomena which depend in an equal or greater degree on many others."

It will not now be difficult to understand the fallacy of *false analogy,* since we have discussed in Chapter V the nature of analogical argument. The fallacy incident to analogy may arise either from the use of metaphorical language or from the failure to distinguish between the essential and the inessential in the properties which are made the ground of inference. Different stages of education are wrongly compared with the steps of a ladder. It is wrongly argued from analogy that just as an individual has birth, growth and decay, so the state, as an organism, has also birth, growth and decay. The similarity upon which the argument is based is not essential. The organic nature of the state is not the same as the organic nature of the body. It is also wrongly argued from analogy that since the paternal form of government is good for the family, it is also good for the state. The nature of the state is not essentially the same as the nature of the family. It was wrongly argued that since seven is a perfect number, and there are seven metals, the heavenly bodies, which are perfect, must also be seven in number. There is really no essential connection between the number seven and perfection. It is also wrongly argued that since our own planet is inhabited by men, other planets also are similarly inhabited. There may not be any essential connection between being a planet and being inhabited.

The fallacy incident to analogy

In classifying objects we should bear in mind that things should be brought under the same class only when they resemble each other in essential points, but not otherwise. False classification is often due

to the misuse of terms. Furthermore, classes should be arranged from the lowest to the highest in a graduated and orderly series, otherwise there may be false classification. Classification by type may lead to false classification, since it is very difficult to find a type, and also because individuals deviate more or less from the type.

The fallacy incident to classification

We know that *explanation* involves generalisation. So if generalisation is false, explanation is also false. Ordinary men generalise wildly. Empirical generalisations, made to explain phenomena, can offer but imperfect explanations. Empirical generalisations are often falsely extended to adjacent cases. An economic or political generalisation which is true of one country may not be true of another country. It is also wrong to suppose that an empirical generalisation is universal and necessary. Thus, to say that water boils at 100° C. is wrong; it boils at that temperature under the pressure of one atmosphere, that is, the normal atmospheric pressure at the sea-level; up a mountain the boiling point is different. Another source of error is to neglect the mutual interaction of cause and effect. Habits of industry often produce wealth, and again possession of wealth may encourage industry. Study may sharpen the understanding, and again the power of understanding may encourage study. In such cases it is wrong to regard one as the cause and the other as the effect. Men of low intelligence are often guilty of drunkenness, but drunkenness again causes low intelligence. The fallacy of explanation being the same as the fallacy of generalisation, we need not discuss it further.

Fallacies incident to explanation

Illusory explanation

Bain notes three forms of *illusory explanation*, the first of which consists in "repeating the fact in different language, assigning no other distinct yet parallel fact". If, to explain why opium causes sleep, we say that it does so because it has soporific virtue, or again if we try to explain why the future must resemble the past by saying that it does so because Nature is uniform, we commit the fallacy of illusory explanation. Another form of it is to "regard phenomena as simple because they are familiar". The fact that water dries up is simple to an ordinary man, but it is not so to a scientist. The lighting of a fire by contact with a flame, writes Bain, is a difficult scientific problem, though to an ordinary man it is a simple fact. The third form of illusory explanation consists in trying to explain a primary law by deducing it from a still more general law. Since mystery surrounds the law of gravitation, men have tried to explain it by referring it to some more general law.

Non-Logical or Material Fallacies

Preliminary remarks

Non-logical fallacies are sub-divided into (a) *Premise unduly assumed* (*Petitio Principii*) and (b) *Irrelevant conclusion* (*Ignoratio Elenchi*). Mill discusses these fallacies under the head 'Fallacies of Confusion'. We should note that Petitio Principii and Ignoratio Elenchi are properly speaking fallacies incident to method.

Petitio Principii

Petitio Principii or Begging the Question includes among its forms what is known as Reasoning in a Circle.

Petitio Principii defined

By principium Aristotle means a self-evident truth. The fallacy of petitio principii occurs when a premise is assumed even though its truth has to be proved. Thus petitio principii in the Aristotelian sense is an undue assumption of axioms. According to Archbishop Whately, petitio principii is that form of fallacy "in which the premise either appears manifestly to be the same as the conclusion, or is actually proved from the conclusion, or is such as would naturally and properly so be proved". Even highly educated men commit this fallacy, for everyone holds a great number of opinions uncritically. The way to avoid this fallacy is to regard nothing as axiomatic which admits of verification. Aristotle

Five forms of it

enumerates five forms of petitio principii. This fallacy is committed, 1. by assuming the very proposition to be proved; 2. by assuming, when the conclusion is particular, a universal which involves it; 3. by assuming, when the conclusion is universal, a particular involved in it; 4. by assuming piece by piece the proposition to be proved; 5. by assuming a proposition which necessarily implies the proposition to be proved. Of these the first two are of frequent occurrence.

The first form has two sub-forms, hysteron proteron and circulus in demonstrando

The first form of the fallacy is strictly speaking the fallacy of *begging the question*, which has two sub-forms, viz. *hysteron proteron* and *circulus in demonstrando* (reasoning in a circle). When the conclusion and the premise are really the same, the

fallacy of hysteron

Hysteron proteron explained and illustrated

proteron is committed. If it be argued that opium induces sleep because it has a soporific virtue, or if we argue that the volume of a body diminishes when it is cooled, because then the molecules which

constitute it become closer, we commit the fallacy of hysteron proteron. This fallacy is committed in a single step of inference. This fallacy is also committed when a concrete term is explained by an abstract term. Thus if we say that a loadstone attracts iron because it has magnetic power, we commit this fallacy. Again if it be argued that a certain measure is bad because it is an innovation, then the fallacy of hysteron proteron occurs. Similarly if it be argued that a particular measure is progressive because it will introduce a change in the existing system, then there is the fallacy of petitio principii or false assumption. The assumption is that what introduces a change is progressive. If it be argued that since justice and wisdom, which are incorporeal things, exist, therefore incorporeal things are existent, then the fallacy of petitio principii is committed. Hysteron proteron may be symbolically expressed thus: S is P, S is P, therefore S is P; or S is P, S is S, therefore S is P. So hysteron proteron may occur when the argument is in the form of a syllogism. Aristotle gives the following example of it: Every rectilinear three-sided figure has its angles equal to two right angles, Every triangle is a rectilinear three-sided figure, therefore Every triangle has its angles equal to two right angles.

Circulus in Demonstrando.—"When the premise does not assume the conclusion but the conclusion is necessary to prove the premise, the argument is said to be circular." If it be argued that early Teutonic societies were held together by kinship because all societies were so held together originally, then the argument is circular, because the conclusion, Early Teutonic societies were held together by kinship, is necessary to prove the premise, All societies were originally held together by kinship. James Smith committed this fallacy when he argued

Circulus in demonstrando explained and illustrated

that if 22/7 be the true ratio existing between the circumference and the diameter of a circle, all other ratios are wrong; and then, that because all other ratios are wrong, 22/7 is the true ratio. The Schoolmen committed the fallacy of circular argument when they argued that mind always thinks because the essence of mind is to think. In this case also the conclusion is necessary to prove the premise. Argument in a circle may be expressed symbolically thus: M is P, S is M, therefore S is P; S is P, M is S, therefore M is P. The following example from Whately is a typical example of circular reasoning: "Every particle of matter gravitates equally; why? Because those bodies which contain more particles ever gravitate more strongly, that is, are heavier. But (it may be urged) those which are heaviest are not always more bulky. No, but still they contain more particles, though more closely condensed. How do you know that? Because they are heavier. How does that prove it? Because all particles of matter gravitating equally, that mass which is specifically the heavier must needs have the more of them in the same space."

The second mode of petitio principii, which consists in assuming a universal which involves the particular proposition to be proved, is an offence against the principles of method. A universal proposition which requires proof should not be regarded as an axiom for the purpose of deducing a conclusion from it. If it be argued that since all cruel men are cowards, and X is cruel, therefore X is a coward, then the fallacy of petitio principii is committed. In this argument the conclusion is assumed in the major premise, which is not an axiom since it requires proof. We have seen in Part I that Mill and others condemned the syllogism on the ground that it in-

The second form of petitio principii

volves the fallacy of petitio principii because the major premise, which is assumed to prove the conclusion, has to be proved with the help of the conclusion.

The third form

"The third mode, of assuming the particular to prove the universal which involves it...is of the nature of a generalisation from simple enumeration." This fallacy is committed in the argument, found in Aristotle, that slavery is natural, because the neighbouring barbarians, who are inferior to the Greeks in intellect, are the born slaves of the Greeks.

The fourth form

The fourth mode is not an independent mode, but a variety of the first. Aristotle committed the fallacy of petitio principii when he successively assumed that the healing art is knowledge of what is wholesome, and is also knowledge of what is unwholesome, to prove that the healing art is knowledge of what is wholesome and unwholesome.

The fifth form

"The fifth mode is when a proposition which is in reciprocal relation to another proposition is assumed as a means of proving the latter." Aristotle committed this fallacy when he argued from the premise that the side of a square is not commensurable with the diagonal, to prove the conclusion that the diagonal is not commensurable with the side. Similarly if it be argued that A is not the son of B, because B is not the father of A, then this fallacy is committed.

Ignoratio Elenchi

"Ignoratio Elenchi means *proving another conclusion*

Ignoratio elenchi defined and explained

than what is wanted. The name does not literally mean that, but 'ignorance of confutation'." If anyone wants to refute a statement he must prove its contradictory to be true, but if he proves something else he does not know what confutation really requires. So ignoratio elenchi (irrelevant conclusion) is a fallacy of refutation or confutation. The argument by means of which a person attempts to disprove a thesis may be sound in itself, yet it will be an ignoratio elenchi if it fails to disprove the thesis which it aims at refuting. "The fallacy lies in proving what is not the precise conclusion which we are are called upon to prove." According to Aristotle an elenchus is a syllogism "with a conclusion contradictory of the thesis to be refuted". Ignoratio elenchi therefore consisted in arguing beside the mark in refutation. The scope of the fallacy has been extended by modern logicians, and it now includes "all cases in which, instead of the required conclusion, a proposition which may be mistaken for it is established".

Some examples of this fallacy

The error in every case consists in *proving the wrong point*. If someone argues that the teaching of Sanskrit in schools and colleges is useless because it is useless in practical life, he commits the fallacy of ignoratio elenchi, because the object of teaching Sanskrit is to impart knowledge and not to make men skilful in business. So the argument is beside the mark. Sometimes this fallacy is committed when the burden of proof is placed on the wrong side. If a man is accused of a crime, it is the duty of the party that brings the charge to prove his guilt. If the accused is asked to prove his innocence, then the burden of proof is placed on the wrong side, and consequently the fallacy of ignoratio elenchi is committed. It is the duty of the person who makes an

assertion to prove its truth; if he asks his opponent to disprove it, then he commits the fallacy of ignoratio elenchi. Here also the burden of proof is placed on the wrong side. Again this fallacy is committed when it is supposed that if objections can be raised against a thesis, then the thesis is disproved. Against every reform some objections can be raised, but every objection does not prove that the reform is bad. Again if by refuting a part of an argument we suppose that the argument as a whole has been overthrown, we commit the fallacy of ignoratio elenchi. If an illustration is given to clarify an issue but it fails to do so, then the fallacy of ignoratio elenchi is committed.

Joseph gives an interesting example of ignoratio elenchi. When Socrates at the time of his trial was asked to call his wife and children before the judges who tried him, so as to excite the sympathy of the judges by entreaties and tears, he refused to do so, because "his part was to persuade them, if he could do it, of his innocence and not of his sufferings". An appeal such as Socrates declined to make is sometimes called argumentum ad misericordiam, which is a form of ignoratio elenchi. We shall now consider some other important forms of this fallacy, which has long been recognised by logicians.

Argumentum ad hominem or *Tu Quoque*.—This fallacy is committed when "being called upon to confute an allegation, I prove something instead about the person who maintains it". If one argues that X is a liar because he drinks, he commits this fallacy. This is a clear case of ignoratio elenchi, because the conclusion is irrelevant to the premise. A drunkard need not be a liar. Statesmen often commit this fallacy when they try to condemn the policy of a man by making allegations against his character. Lawyers often

resort to this kind of argument to achieve their object. Suppose we argue that the policy of a particular man is bad because it is not consistent with his old policy, then we commit this fallacy. I commit this fallacy "if I condemn Home Rule for Ireland on the ground that Parnell was an adulterer''.

Argumentum ad baculum.—This is a sub-form of argumentum ad hominem. This fallacy is committed when to refute the argument of an opponent we resort to force, failing to refute it by a counter-argument. If we shout down an opponent when he speaks before an audience, we commit this fallacy. Similarly if we silence a man at the point of a bayonet we commit this fallacy. Mr. Stock remarks, "To knock a man down when he differs from you in opinion may prove your strength, but hardly your logic."

Argumentum ad populum.—This fallacy is committed when appeal is made to popular passion or prejudice to prove a thesis. Demagogues often try to command the support of their audience by flattering them and appealing to their vanity. To rouse the passion of a mob, appeal is sometimes made to their religious susceptibilities. If a mother argues against the utility of education by stating that her son would not have died if his body had not been weakened by study, she commits this fallacy.

Argumentum ad ignorantiam.—This fallacy is committed if by taking advantage of the ignorance of a man we try to make him accept a statement which is by no means proved. This is closely allied to the argumentum ad populum. Educated men sometimes take advantage of the ignorance of ordinary men to prove or disprove some thesis.

Argumentum ad verecundiam.—Argumentum ad ignorantiam is often allied to Argumentum ad verecundiam. Argumentum ad verecundiam consists in proving or disproving a thesis by appealing to authority. This form of ignoratio elenchi is of frequent occurrence. Medieval thinkers often tried to prove or disprove something by appealing to the authority of Aristotle. If we ask someone to accept a statement or to advocate some measure because some eminent men did it in the past, we commit this fallacy. Thus many argued against the heliocentric theory on the authority of Aristotle. If we argue that the world originated in such and such a way because the Vedas or the Bible support it, then we commit this fallacy.

Non-Causa Pro Causa

By causa we must not understand in this case causa essendi (material cause), which is the usual sense of the term; the term here means causa cognoscendi (sufficient reason). This fallacy is committed when sufficient reason is not provided to prove something. It is therefore not an inductive fallacy. It is concerned with the justification of a proposition and not with the determination of a fact. In mistaking for a cause or reason what is not a cause we commit this fallacy. This fallacy is committed when the principle of sufficient reason is violated. This fallacy is generally regarded as incident to reductio ad absurdum. If it be argued that the world cannot be flat because such a world would be infinite, and an infinite world could not be circumnavigated, but the world has been circumnavigated, then this fallacy is committed, because here the impossibility of circumnavigation of the world is not due to the flatness of the world, but to its being infinite. If the world were flat but finite, it could be circumnavigated. So the thesis of the

world's being flat is unfairly discredited by the above argument. It is a reductio ad absurdum. This fallacy is often identified with *post hoc ergo propter hoc*, the nature of which we have previously explained. But *non causa pro causa* and *post hoc ergo propter hoc* are not really the same.

Non Sequitur

This fallacy is committed when the conclusion does not necessarily follow from the premises. "Both the premises and the conclusion may be granted, and yet the derivation of the one from the other denied; or the premises may be accepted, and the conclusion ostensively drawn from them rejected." If it be argued that in India poverty has increased after political reforms have been introduced there, and therefore such reforms are undesirable, we commit this fallacy, because increase of poverty may not be due to the introduction of reforms. This fallacy is also known as *non propter hoc*. Welton and Monahan give the following example of this fallacy:—"If it be argued that the increase of schools has been evil, and for proof it be pointed out that official returns show a continuous increase of crime, we have an obvious non-sequitur." This fallacy is often the same as the fallacy of four terms, and it occurs when the thesis is not connected with the conclusion by any middle term. (The fallacies known as post hoc ergo propter hoc, non-causa pro causa, and non sequitur, are hardly distinguishable. They are almost the same.)

EXERCISES

CHAPTER I

DEFINITION, NATURE AND SCOPE OF INDUCTION

1. Show that deductive reasoning can give material truth only when it is aided by induction. Can the truth of a syllogistic reasoning be proved by pro-syllogisms? Illustrate your answer.

2. Discuss whether induction is prior to deduction or deduction to induction. In what way is deduction dependent upon induction? Distinguish between formal validity and truth. Illustrate your answer.

3. What is meant by saying that induction is an inverse process? Is this view correct? Are both induction and deduction natural processes?

4. Define induction and clearly explain its nature.

5. Show how scientific induction differs from induction by simple enumeration, analogy, and probability. Illustrate your answer.

6. Clearly distinguish between induction proper and processes of reasoning improperly called inductive.

7. In what sense does inductive reasoning depend upon similarity? Is inductive reasoning a passage from the particular to the general or from the particular to the particular?

8. Clearly explain the statement that the conclusion of an induction is a universal, necessary and real proposition.

9. Can there be a legitimate induction from the observation of one instance only? Discuss the question fully.

10. Is it necessary in induction to observe a large number of instances? What is the basis of induction by simple enumeration?

11. Can an induction be regarded either as a notion or as a definition?

12. Is an induction the same as colligation of facts?

Fully discuss the controversy between Whewell and Mill on this point.

13. Is mathematical reasoning inductive? Clearly explain the nature of mathematical reasoning.

14. What is perfect induction? Is it induction proper? Illustrate your answer. Show with illustrations that some mathematical reasonings are analogous to perfect induction.

15. "What is called perfect induction is nothing but deductive reasoning." Explain this statement by reducing a perfect induction to a syllogism. What is meant by induction by parity of reasoning?

16. Can induction proper be syllogistically expressed? Discuss in this connection Whately's and Mill's views of inductive syllogism.

Hints.—(1) **Aristotle** is said to have tried to reduce Induction to the Syllogism in the following way (although what he actually sought to do was something else, as will be clear as we proceed):

Man, horse, mule, etc., are long-lived creatures.
Man, horse, mule, etc., are all the bile-less animals.
∴ All the bile-less animals are long-lived creatures.

This is a syllogism in the third figure.

Aristotle called it '*proving the major term of the middle by means of the minor.*' Here the terms are to be taken in their denotation, and not in their usual sense. The predicate of the conclusion (*long-lived creatures*) has the widest extent, and so it is to be taken as the *major term.* The subject of the conclusion (*all the bile-less animals*) is found to have a medium extent when it is compared with the major term and the other, *viz.*, 'man, horse, mule, etc.' So it is called here the *middle term.* And the term (*man, horse, mule, etc.*) that occurs as subject in both the premises has the least extent, and is therefore called the *minor term.* Taking the terms in this way, we find that the above syllogism purports to prove the major term (*long-lived creatures*) of the middle (*all the bile-less animals*) by means of the minor (*man, horse, mule,* etc.). That is to say, the above syllogism seeks to prove that the fact of being *long-lived creatures* is true of *all the bile-less animals* by showing that the fact of being *long-lived creatures* is true of *man, horse, mule, etc.*

This so-called Inductive Syllogism, it is easy to see, has many peculiarities:

(*a*) It is, strictly speaking, not a syllogism at all. It is apparently in the third figure, since the middle term is subject in both the premises. But from the combination AA in the third figure we have I for the conclusion, as in *Darapti*, and not A, as is the case here. So one might condemn it as involving the fallacy of *illicit minor*.

(*b*) But then the minor term is distributed here, being the predicate of an U proposition, and not of A. An A proposition does not distribute its predicate, but an U proposition does. Judged from this viewpoint, the syllogism cannot be said to involve the fallacy of illicit minor, and is all right as far as it goes.

(*c*) It does not rest on Aristotle's own famous *dictum de omni et nullo*, which lays down that whatever can be predicated of the whole, can be predicated of the parts. It seems to rest, but actually speaking does not do so, on a reversal of that famous dictum, which would be: Whatever can be predicated of the parts, can be predicated of the whole. Actually it rests on the principle that whatever can be predicated of *all* the parts, can be predicated of the whole. So the question whether all the relevant instances have been examined—a question that has been raised by many—does not arise at all. It *is* a case of observing *all* the relevant instances. So the argument is entirely flawless. Aristotle is not so easy to refute as many, in their ignorance, take him to be.

Nor does Aristotle himself look upon the Inductive Syllogism as a kind of proof, distinct from Deduction. On the contrary, he is definitely of the opinion that, the so-called Inductive Syllogism is "a mode of arranging a deductive argument so as to enable us to realise psychologically the truth of the general principle, which is the real major premise—a mode of illustrating the principle by bringing forward instances."

Modern criticism, however, rejects Aristotle's Inductive Syllogism as a case of Perfect Induction. It fails to take the inductive leap, say its critics, which is the distinctive mark of what is known as Scientific Induction.

(2) **Aldrich** and **Whately** tried to reduce Induction to the following form of syllogism:

The men I have observed *and the men I have not observed* are mortal.
All men are the men I have observed and the men I have not observed.
∴ All men are mortal.

This is a valid syllogism in Barbara. It is claimed in its behalf that, although a syllogism, it represents correct Scientific Induction, as it establishes a general proposition on an observation of some particular cases. It involves the inductive hazard, it is said, that is lacking in Aristotle's so-called Inductive Syllogism; for it takes a leap from the known to the unknown.

But what is the ground of the major premise here? One might as well observe some men, and assume that, the men one has observed and the men one has not observed are all white men. The so-called Inductive Syllogism of Aldrich and Whately, it is clear, assumes the very thing it seeks to prove. Bain says, "The major here obviously assumes the very point to be established, and makes the inductive leap. No formal logician is entitled to lay down a premise of this nature. The process altogether transcends syllogism or formal logic." How to establish the major premise in such a case is the very problem of induction.

But perhaps Aldrich and Whately are not to be dismissed so lightly as that.

(3) **Mill** justifies the passage from the known to the unknown on the ground of the *Uniformity of Nature,* and says that, 'every Induction may be thrown into the form of a syllogism by supplying a major premise,' which, according to him, would, in every case, be the principle of 'the uniformity of the course of nature.' Thus, according to Mill, an Inductive Syllogism would have the principle of the Uniformity of Nature as its major premise, and the statement of the facts observed as the minor premise, and would therefore be of the following type:

What is true of some members of a species under certain conditions is true of all the members under the same conditions;
Mortality is true of some members observed, such as John, Peter, Henry, and others, of the species

man under certain conditions, such as the presence of human nature in them, etc.

∴ Mortality is true of all members of the species man under the same conditions, that is to say, all men are mortal.

More simply the syllogism may be stated thus:

What is true of John, Peter, Henry and others, is true of all men.

John, Peter, Henry, and others are mortal.

∴ All men are mortal.

Ordinarily Mill's so-called Inductive Syllogism is taken to be satisfactory. But there are reasons to doubt if it is actually so. Mill himself believes that the principle of the Uniformity of Nature, which, according to him, must form the major premise of all inductive syllogisms, is only a generalisation from experience. If it is so, it must lack the certainty we rightly expect in the major premise of a syllogism, and Mill can scarcely escape the censure that he is defeated at his own game.

17. Show that inference involves selection, comparison and discrimination.

18. Explain the characteristics of scientific thinking.

19. Explain the significance of the statement that the universe is orderly and laws or forms exist in natural things.

20. Explain how sciences employ deductive or inductive method or both methods. Why is induction supposed to be the scientific method par excellence?

21. Is induction a science of discovery or of proof? Discuss the controversy between Whewell and Mill over this question.

22. Explain the form of reasoning, deductive or inductive or both, implied in the following propositions, indicating the premise or conclusion left unexpressed, and estimate the value of the reasoning:—(*a*) The sun will rise to-morrow morning.

(*b*) The lower animals feel pain just as we do.

(*c*) He will die in a few hours.

(*d*) Intermittent fever is found only in places where there are marshes even though they differ in every other respect.

(*e*) The inner world of mind attains the light of

knowledge through seven organs of sense, therefore some medieval astronomers said there must be seven planetary bodies to illuminate the outer world of Nature.

(*f*) The factory commissioners say in their report, "the past and present conditions of work in factories are undoubtedly calculated to cause physical deterioration; and we are struck with the marked absence of elderly men among the operatives."

[**Hints.**—(*a*) This is a case of Induction per Simple Enumeration. The conclusion is based on uncontradicted past experience.

To an astronomer, however, it is, to a large extent, a case of mathematical reasoning, which is essentially deductive in character; but as it is not a case of pure mathematics and has to rely on the observation of certain heavenly phenomena, it has an inductive flavour about it, but nothing more than that.

(*b*) This is a case of argument from analogy. What we can observe in animals are certain patterns of behaviour analogous to our own. Argument from analogy is ordinarily supposed to have certain things in common with induction.

(*c*) This is a case of argument from probability, which rests on calculation of chances, and is ordinarily supposed to be an inductive procedure.

(*d*) The reasoning is based on what is known as the Method of Agreement. (See Chapter VI).

(*e*) Bad analogy.

The rest to be done by the student.]

23. (*a*) Induction is the process of establishing general propositions, and deduction is of interpreting them. Explain and illustrate this. (*b*) Is the theory of reasoning here implied admitted by all logicians? If not, what other theory has been held?

24. You draw an isosceles triangle on a board, and prove that its two basal angles are equal, and then draw the conclusion that all isosceles triangles have their basal angles equal. Explain the logical character of this argument.

25. What is meant by demonstration? What kinds of inference are of demonstrative character, and what kinds are

merely probable? Explain the reason in each case, and give examples.

26. State, explain and illustrate the various kinds of inductions improperly so called. Explain clearly in each why it is not an induction in the proper sense of the word.

27. Name the principal deductive and the principal inductive sciences, stating in each case your reason for considering it to be deductive or inductive. A science at one time wholly inductive may become at another time more or less deductive. Explain this.

28. Can we form a valid universal proposition about facts if we have not actually observed all the individuals signified by the subject of the proposition? If so, how?

29. Determine the character of inference, and show how it is illustrated in induction.

30. What are the characters of a valid induction? Explain and illustrate them fully.

31. Explain the terms Perfect Induction, Imperfect Induction, Complete Induction, Incomplete Induction, Proper Induction and Improper Induction.

32. What are the different kinds of process that simulate induction? Exhibit and illustrate each of them, and explain in each case why the process is not real induction.

33. Is inductive reasoning merely the converse of deductive reasoning? Fully discuss the question, and in this connection bring out clearly the relation of the one to the other.

34. Distinguish between perfect and imperfect induction, and discuss the question whether perfect induction is demonstrative and syllogistic while imperfect induction is neither.

35. "Induction is the inverse process of deduction." Examine this statement. Exhibit the true relation between the two processes of reasoning.

36. What are the marks of inductive inference? How does induction differ from colligation of facts?

37. "The difference between deduction and induction is not one of principle, but of starting point." Discuss.

CHAPTER II.

POSTULATES OF INDUCTION

Exercises

1. Why is the uniformity of causation said to be the formal ground of Induction? Can we say that the Uniformity of Nature and the Principle of Causation are the formal grounds of induction?

2. What are the postulates of induction and why are they so called?

3. What are the fundamental beliefs of the scientists? Is scientific investigation possible without these beliefs?

4. Explain what is meant by saying that the universe is identical, persistent, continuous and simple.

5. Fully explain the principle of the Uniformity of Nature. State and illustrate its different forms.

6. Elucidate the statement that the world is not only uniform but also multiform.

7. Can we regard Nature as uniform even when it abounds in irregularities?

8. Why is it that a single instance may justify us in making an inductive generalisation, though at other times a number of instances may not do so?

9. What is meant by 'Paradox of Induction'? Is Mill justified in holding that the ground of induction is itself an induction?

10. What is meant by co-existence? What are its different forms? Illustrate your answer.

11. State and explain the principle of Universal Causation. Is it to be regarded as the ground of induction?

12. Why do we speak of causal laws? How are they related to the principle of Universal Causation?

13. Explain with illustrations Aristotle's distinction between four kinds of causes. Is this view of causation scientific?

14. Explain Hume's view of causation that cause is an invariable antecedent. How does Mill modify it?

15. Explain the statement that to understand the problem of causation is to understand the problem of induction.

16. Define 'Cause' after Mill, and distinguish Mill's view of causation from the popular or common sense view.

17. Does the cause produce the effect? Clearly explain and examine the statement that the cause is an invariable and unconditional antecedent to the effect.

18. Explain the terms agent, patient and collocation.

19. What according to you is the scientific view of causation? Discuss the question fully.

20. What is meant by plurality of causes? Is the doctrine of plurality of causes sound?

21. What is meant by saying that causal relation is one-one and not many-one?

22. What does Mill mean by permanent causes? Can the same cause produce different effects? Can causal relation be one-many?

23. What is the principle of conservation of energy? Does the conception of causation undergo any modification when viewed from the standpoint of conservation of energy?

24. Distinguish between potential and kinetic energy. Why, from the viewpoint of conservation of energy, are cause and effect, regarded as quantitatively equivalent? Can such equivalence between cause and effect be established in every case of causation?

25. Does the conception of conservation of energy modify the conception of causation in any way?

26. Distinguish between homogeneous and heteropathic inter-mixture of effects, with examples.

27. (i) If it be true that the same cause produces the same effect, does it follow that the same effect is always produced by the same cause? Give your reasons for your answer, and support it by illustrations. (ii) Show how the principle involved here gives rise to difficulty in drawing inferences, giving examples. How may the difficulty be overcome? Give examples.

28. What do you consider to be the difference between cause and condition? Give examples. If a workman carrying a burden falls from a ladder and is killed, what do you

consider to be the cause and what the conditions of his death, and why? A distinction may be made between cause from the scientific and cause from the merely practical point of view. In the above case, what may be regarded as the cause from the merely practical point of view?

29. (i) Explain and illustrate fully the principle of the uniformity of Nature. (ii) What are the fundamental kinds, classes or branches of uniformity found in Nature? (iii) what do you consider to be the ground or evidence underlying the belief in uniformity? (iv) What is meant by saying that uniformity is the ground of induction? (v) Do you consider cyclones and earthquakes to be consistent with uniformity?

30. A great part of the knowledge of every individual is derived not directly from inference, nor even from perception, but from Authority. What part of your knowledge have you derived from Authority? On what considerations does the value of Authority mainly depend?

31. (i) What is meant by the cause of an event? (ii) Explain the difference between the cause and the conditions of an event. (iii) Distinguish between proximate and remote causes. Illustrate your meaning by examples. (iv) A man is crossing the river in a small boat, a sudden squall of wind comes on, the boat founders and the man is drowned. What do you consider to be the cause and the conditions of his death?

32. Why is it that one should not regard night as the cause, nor even as a universal condition of day? Explain cause and condition.

33. What do you understand by plurality of causes and the mutuality of causes and effects? Illustrate your answer by examples.

34. State and explain the grounds of inductive inference.

35. "The ground of induction is itself an induction." Fully discuss this. How can a conclusion which asserts more than the premises do be valid? Fully discuss the question.

36. Explain and illustrate composition of causes. How does it differ from heteropathic inter-mixture of effects?

37. A man goes out into the open air where a cold breeze is blowing and gets a cold. What is the cause of his getting a cold from the practical point of view, and from the

scientific point of view? Fully explain the scientific conception of causation.

38. "I see my brother." How far is this affirmation based on observation and how far on inference? Show by examples how experiments help to prove causation.

39. Explain the conception of a cause as a group of antecedents necessary to and sufficient for the effect. What is meant by negative condition of an event?

40. "The cause is the invariable and unconditional antecedent." "The cause is the sum of conditions negative and positive." Explain and illustrate either of the above statements.

41. Can an effect be produced by alternative causes? Explain and illustrate the different modes in which two or more causes combine to produce a single effect.

42. Explain and analyse the conception of the uniformity of Nature, and show how it forms the foundation of inductive reasoning.

43. What is the presupposition of induction? Mention the different ways in which it has been formulated. Which of them do you think to be most adequate, and why.

CHAPTER III.

OBSERVATION AND EXPERIMENT

Exercises

1. Explain clearly the nature of observation and experiment. Why are they called the material grounds of induction?

2. Clearly explain the nature of observation. What are the conditions of correct observation?

3. Distinguish between non-observation and mal-observation. Illustrate your answer.

4. Give some examples of discoveries made by scientific observation.

5. Show that correct observation and experiment largely depend upon previous knowledge and insight.

6. How do instruments aid observation? Illustrate your answer. Can everyone use scientific instruments for the purpose of right observation?

7. Explain how observation involves selection, analysis and inference.

8. Compare observation with experiment. Is the difference between them fundamental? Explain their respective advantages.

9. Are all sciences experimental? Why are experimental sciences progressive?

10. Explain clearly how experiment enables us to establish reciprocal relation between cause and effect.

11. How does testimony contribute to the progress of science? Should we accept all testimony? On what conditions does the value of testimony depend?

12. Explain why the scientific enterprise is a co-operative enterprise.

13. Clearly explain the nature of inductive procedure.

14. Define observation and experiment, giving examples of each, and explain why these processes require treatment in inductive logic. What are the advantages of the latter over the former? What sciences depend mainly on observation and why?

CHAPTERS IV-V.

HYPOTHESIS: SUGGESTION OF HYPOTHESIS

Exercises

1. Define hypothesis and clearly explain its nature. What is its place in induction? Compare hypothesis with induction.

2. Distinguish between popular and scientific hypothesis. Should we, in science, explain phenomena by ultimate purpose? Give some examples of legitimate hypothesis.

3. Discuss whether Mill's or Whewell's attitude towards hypothesis is justified.

4. Explain the terms—law, hypothesis, fact, and theory.

5. What are the different forms of hypothesis? Explain them with illustrations. Explain the terms—working hypothesis and analogical hypothesis. Is hypothesis useful in science? Can any strict line of demarcation be drawn between hypothesis about law and hypothesis about cause?

6. State and explain the conditions of a valid hypothesis.

7. Clearly explain the statement that a hypothesis should be a vera causa.

8. Explain with illustrations—experimentum crucis and crucial instance. What is consilience of induction?

9. Explain the rule that the law of parsimony should be observed in forming hypotheses. What is meant by the principle of simplicity in this connection?

10. How do you prove a hypothesis? Distinguish between deductive and inductive proof of hypothesis.

11. Is hypothesis an abstraction? Fully discuss the question.

12. What was the attitude of Newton towards hypothesis? Was he an enemy of hypothesis? Why did he say 'Hypotheses non fingo' (I do not make hypotheses)?

13. Clearly explain the utility of hypothesis in science.

14. What is a hypothesis? Give an example from common life. Explain the use of hypothesis. What are the conditions of a good hypothesis? Suppose that on returning home you find one of the panes of your window broken. Show how would you apply the method of hypothesis in this case.

15. Explain the use of hypothesis in scientific investigations. Given a verifiable hypothesis, what constitutes its proof or disproof? Distinguish between a working hypothesis and an established hypothesis.

16. Distinguish between a theory and a hypothesis. Give the canons to which a good hypothesis must conform, and illustrate them. Explain the functions of hypothesis in induction.

17. What are the circumstances favourable to discovery? What are the different forms of hypothesis?

18. Do hypotheses assist observation in any way? If so, how? What are the other uses of hypothesis? Distinguish between a working hypothesis and a descriptive hypothesis.

19. State and explain the essential conditions of a valid hypothesis. If there has been a theft in a room, how would you proceed to frame hypotheses, (a) as to the identity of the thief and (b) as to the manner in which the theft was committed?

20. Explain the law of parsimony in connexion with hypothesis.

21. What are the processes of thoughts which suggest hypotheses? Briefly explain their nature.

22. Clearly explain the nature of induction by simple enumeration, comparing it with scientific induction and perfect induction. Illustrate your answer.

23. "Empirical laws are inductions by simple enumeration." Explain this statement and give some examples of empirical laws. How do such laws differ from scientific laws proper.

24. Is induction by simple enumeration induction proper? State your reasons. Why is it so named? Compare it with analogy.

25. Can causation be regarded as an empirical law? Give reasons for your answer.

26. What is the logical character and value of induction by simple enumeration?

27. Why is classification regarded as an early form of induction by simple enumeration? Can induction by simple enumeration lead to scientific induction?

28. Define analogy and clearly state its nature. Is it induction proper?

29. Why is it said that men often pass from induction by simple enumeration through analogy to scientific induction? Why is it said that while simple enumeration rests upon the counting of instances, analogy weighs them.

30. Compare analogy with scientific induction. Illustrate your answer. What are the different forms of analogy? Exemplify them.

31. Show with examples that some arguments from analogy are childish while others are plausible.

32. Distinguish between positive and negative analogy, and between implying and implied property. Illustrate your answer.

33. What are the conditions upon which the value of analogical arguments depends? Is Mill's view in this regard satisfactory?

34. "The value of an analogical inference depends on the degree as well as on the kind of resemblance." Show by examples how one kind of resemblance may be more important than another as the ground of inference.

CHAPTER VI

MILL'S EXPERIMENTAL METHODS

Exercises Worked Out

1. *Scarlet flowers have no fragrance.*

This conclusion may be said to be based on the *Method of Agreement*, which requires several cases, at least two, with a particular antecedent and a particular consequent common to all of them. Here the common antecedent is the scarlet colour of flowers, and the common consequent, the absence of fragrance in them. All the cases examined differ in all other respects. Hence these two (scarlet colour and absence of fragrance) are supposed to be causally connected. The scarlet colour is believed to be the cause of the absence of fragrance in flowers. Hence the conclusion.

But it may well be a case of mere co-existence, and not of causation. Moreover, the best that can be said of it is that it is only probable. The Method of Agreement cannot conclusively establish a causal relation, or even a case of mere co-existence.

But why so much fuss over experimental method and all the rest of it? It might as well be said to be a case of

Induction by Simple Enumeration, which consists in arriving at a general real proposition on the ground of uncontradicted experience, without any attempt at explaining a causal connexion. Ordinarily a statement like the one we are here concerned with makes little or no reference at all to a causal connexion, but merely states a conclusion that, to all intents and purposes, is universal, real and synthetic. In a like manner, until the black swans of Australia were discovered, peoples of the northern hemisphere had, under all manner of conditions, seen only white swans that differed amongst themselves in all respects except two, *viz.*, in being swans and in being of a white colour. This led them to conclude that swans were white, and from this one might have as well argued that being a swan (antecedent) was the cause of having white colour (consequent), and so the two were related by way of cause and effect.

Actually speaking, the conclusion that scarlet flowers have no fragrance is a case of simple enumeration. No question of any application of the experimental methods arises here at all. Logic has often led many to the most astounding illogicalities.

2. *If a particular portion of the brain is removed, a particular part of the body is paralysed.*

Valid. It is based on the *Method of Difference*, which requires only two instances that agree in all respects except one. Here in one instance, the whole brain and the normal functioning of all the parts of the body go together; while in the other, the removal of a particular portion of the brain (antecedent) is followed by paralysis of a particular part of the body (consequent). Hence the conclusion that the two are causally connected—the removal of that portion of the brain is the cause of paralysis of that part of the body.

3. *Intermittent fever is found only in places where there are marshes, even though they differ in every other respect.*

Valid. The *Joint Method* has been employed. This method requires two sets of cases, a positive set and a negative set. Each set must contain at least two instances, preferably more. The positive set should contain some particular antecedent and some particular consequent in all the instances contained in it. In the negative set, this parti-

cular antecedent and the particular consequent must be wholly absent.

There are two sets of instances here. In the positive set, all the instances point to marshes (antecedent) and intermittent fever (consequent), while in the negative set we find neither marshes nor intermittent fever, that is to say, we find that where there are no marshes, there is no intermittent fever. Thus we come to conclude that marshes are the cause of intermittent fever.

4. *Cocoanut trees best flourish in places not far removed from the sea.*

Valid. The conclusion has been arrived at by the application of the *Method of Concomitant Variation*, which is applied when the phenomenon under investigation is found to vary along with another circumstance. The nearer it is to the sea, the better is the growth of the cocoanut tree; the further it is from the sea, the worse is its growth. Hence the above conclusion.

It may as well be a case of the application of the *Method of Agreement*. One might arrive at the same conclusion on the observation of a large number of instances.

5. *Water is jointly conveyed into a tank by three pipes of unequal size at the rate of* 10 *gallons per minute. It is known that the first two pipes admit water at the rate of* 7 *gallons per minute. Therefore the amount of water admitted by the third pipe is at the rate of* 3 *gallons per minute.*

Valid. *The Method of Residues* have been applied here. This method is applied when a part of complex effect is known to be due to some factor or other, and one is required to find out the cause of the remaining portion of the whole effect. Here the total effect is known to be a storage of 10 gallons of water per minute; it is also known that a part of this total, *viz.*, 7 gallons of water per minute is due to the first two pipes. From these data, it is concluded that the residuum, *i.e.*, the remaining 3 gallons per minute, is due to the third pipe.

6. *The weight of the load is the total weight less the weight of the cart.*

Valid. This, too, is based on the *Method of Residues*. The complex effect, *i.e.*, the total weight is, in a sense,

known; it is x. The weight of the cart is also known, in a like manner, to be y. Hence the weight of the load is $x—y$.

7. *Cold applied to water in an iron vessel freezes it. Cold applied to cocoanut oil in a glass bottle freezes it. Therefore cold is the cause of freezing.*

Valid, so far as it goes. *Method of Agreement* applied. The two cases observed disagree in all respects except one, *viz.*, in the application of cold (antecedent) and the freezing of liquids (consequent). Hence cold is the cause of freezing.

N. B. The Method of Agreement is pre-eminently a method of observation, and not of experiment. But here it is concerned with experiment. Anyway, the number of instances is too small to justify a conclusion even experimentally.

8. *Heat is the cause of the melting of ice.*

Valid. *Method of Difference* applied. Two cases, which resemble one another in all respects except one, have been observed, presumably in the laboratory under controlled conditions. In one case the presence of heat (antecedent) is found to be followed by the melting of ice (consequent), while in the other, neither heat nor the melting of ice is present at all. Hence the above conclusion.

9. *A large number of birds have been observed, and found to be without teeth. Hence the conclusion that birds have no teeth.*

If any of the experimental methods has been applied to this case, it must be the *Method of Agreement.* All the birds observed agree only in having a particular antecedent and a particular consequent common to them all, *i.e.*, in being birds (antecedent) and in being toothless (consequent). Hence the conclusion that birds have no teeth.

But being birds and having no teeth may as well be a case of mere co-existence, and not of causation.

It may also be a case of Simple Enumeration.

10. *The increase in the number of crimes in a village is due to the removal of the police station.*

The conclusion can be justified only on the assumption that nothing else in the village changed with the removal of the police station except the renewed activities of criminals. The argument then would be based on the *Method of Differ-*

ence, as both the cases, *viz.*, the village with the police station and without it, would agree in all particulars except one, *viz.*, the removal of the police station (antecedent) and the increase in crimes (consequent).

But it is too much to expect that, simultaneously with the removal of the police station, nothing else had changed. So it is a case of a wrong application of the Method of Difference.

Exercises

1. Give the names of Mill's inductive methods. Show that their function is to eliminate irrelevant circumstances.
2. Show that the canons of Mill can be deduced from the principles of elimination, and ultimately from the principles of causation.
3. Which method of Mill's do you regard as fundamental and why? Do they suggest hypotheses or prove them?
4. State and explain with concrete examples the method of agreement. Can it successfully eliminate irrelevant circumstances and prove causal connection? Explain the merits and demerits of the canon.
5. State the method of difference and explain it with a concrete example. Can this method prove causal connection? What are the merits and demerits of this method?
6. State the joint method and explain it with a concrete example. Can this method prove causal connection? What are the merits and defects of this method? When is this method applicable?
7. State the method of concomitant variations and explain its nature with a concrete example. When is this method specially applicable? Can it prove causal connection? Why is it called the graphic method? Explain the merits and defects of this method.
8. State the method of residues and explain its nature with an example. Is it an inductive method? When is this method specially applicable? What are the merits and defects of this method? Is it a method of discovery or of proof?

9. Which of the five methods are most fundamental, and why?

10. Do the canons of Mill fulfil their purpose? Critically examine the methods.

11. Can we regard these methods as unconnected as Mill supposes? Are the canons of Mill indispensable for inductive sciences?

12. What is meant by varying the circumstances? What instruments are used for this purpose? Do they subserve the purpose?

13. To what extent are the direct methods successful in establishing causal connection? Why are the canons of Mill called direct methods? How do they differ from the deductive method?

14. When are inductive methods applicable? Can there be rules for scientific enquiry?

15. Give two examples of scientific inductions, clearly pointing out the methods by means of which the conclusions have been finally established.

16. State fully and clearly in your own words the method of concomitant variations, with examples. On what canon or principle is it based? Of what other method is it a modification? Is it a method of observation or of experiment or of both? In what class of cases is it the only possible inductive method, and why?

17. Why are the methods of agreement and difference regarded as the methods of observation and of experiments respectively.

18. State in your own words, and illustrate with examples, the method of difference. Show by means of common instances that the method plays a great part in everyday inferences. Suppose that wherever there are anopheles mosquitoes, there is malaria, but that malaria is found also where there are no mosquitoes: what conclusion can you draw from this?

19. Explain the method of agreement. Give examples. How is the method frustrated? Give an example. What is the remedy?

20. Explain and illustrate any four of the following:- Varying the circumstances, Inductive elimination, Plurality

of causes, Intermixture of effects, Law of Nature, Empirical law.

21. When is it necessary to employ the method of concomitant variations? Explain and illustrate the method, indicating its different forms.

22. Is elimination the essence of induction? Fully discuss this question. What exactly has elimination to do with the proof of a hypothesis?

23. Explain how plurality of causes and intermixture of effects affect the application of the method of agreement. What advantage has the method of difference over the method of agreement and what advantage has the latter over the former?

24. What are the two main principles involved in Mill's canons of the experimental method? What are the two ways in which the method of residues can be applied?

25. Discuss the question whether the inductive methods may be viewed as mere weapons of elimination. Examine the attempts at reducing them to one or two fundamental methods.

26. Explain, giving a concrete example, the method of difference, and point out its relation to the methods of concomitant variations and residues. Explain the nature of the phenomena for the investigation of which the last two methods are particularly suited.

27. Name the experimental methods by which each of the following conclusions is arrived at:—

(a) If a particular portion of the brain is removed, a particular part of the body is paralysed.

(b) The more a body is heated, the more it expands.

(c) Scarlet flowers have no fragrance.

28. Name the experimental methods by which each of the following conclusions is proved, explaining its applicability in each case:—

(a) Heat is the cause of the melting of ice.

(b) Cocoanut trees best flourish in places not far removed from the sea.

(c) Despotic government gradually disappears as the people become more and more educated.

29. Explain what is meant by saying that the methods of agreement and of difference are mainly methods of observation and experiment respectively. How does the method of difference differ from the method of residues?

30. "The method of agreement is a method of discovery. The method of difference is a method of proof." Explain the significance of the above remark.

31. What do you understand by the experimental methods? Why are they so called? Indicate the use of each.

32. Name the experimental method on which each of the following arguments is based, giving your reasons:—

(a) Two small pieces of blanket, exactly alike in all respects except that one is coloured white and the other black, are placed on a block of ice. After a certain time it is found that the black piece has sunk deeper into the ice than the white one. Therefore it is concluded that black absorbs more heat than white.

(b) A large number of birds have been examined and found to be without teeth. Therefore it is inferred that all birds are without teeth.

(c) A nation becomes more prosperous as it develops in an increasing measure habits of industry and prudence.

(d) One Sunday morning in a poor country parish there appears the surprising phenomenon of a half-sovereign in the offertory. The clergyman knows by repeated experience that none of his flock ever by any chance gives more than a silver three penny piece; but he has perceived a stranger in the congregation and therefore he concludes that he is the donor of the half-sovereign.

33. Name the experimental method on which each of the following arguments is based, stating your reasons in each case:—

(a) Intermittent fever is found only in places where there are marshes, even though they differ in every other respect.

(b) Despotic government gradually disappears as the people are more and more educated.

(c) Able men have generally very bad handwriting, while good handwriting is frequently found in men doing comparatively little mental work. Hence it is inferred that mental strain is the cause of poor penmanship.

(d) Both mosquitoes and cases of malarial fever have, in certain parts of Italy, in West Africa, and elsewhere, become much rarer since the districts have been well drained. Is malarial fever due to the presence of mosquitoes?

CHAPTER VII

MILL'S DEDUCTIVE METHOD

1. Indicate the different forms of Deductive Reasoning, and explain, with appropriate illustrations, the importance of what Mill calls 'the Deductive Method in Induction.'

2. Determine the scope of the Deductive Method in Induction.

3. Explain the relation between the Deductive Method in Induction and Hypothesis.

4. Critically estimate the part deductive reasoning plays in Induction.

5. What do you understand by a Deductive, and what, by an Inductive, Science? Name the principal Deductive and Inductive Sciences, and say why certain sciences are Deductive and certain others Inductive.

[**Hints.**—A Deductive Science proceeds on the assumption of certain principles, and deduces conclusions from them, while an Inductive Science is supposed to go by facts of experience alone, so that it might establish general truths or laws from them. But the distinction is too fine to be of much practical value. Every science has got to take certain things for granted, or it is no science at all. Physics is supposed to be an Inductive Science; but it has to assume a good number of things before it can start on its career of research. It has to assume, for instance, that matter and motion are realities amenable to quantitative treatment, and so on and so forth. So is also the case with Chemistry, another Inductive Science. Botany, Biology, Zoology, in fact, all the so-called Inductive Sciences have to proceed on assumptions of their own, besides those that are accepted by every science. Mathematics, on the other hand, is supposed to be the Deductive Science par excellence. It assumes all manner of numerical and spatial relations, and proceeds to make deductions from them. But pure mathematics apart, much of its generalisations have to be verified

by experience. Mechanics is often spoken of as a Deductive Science; yet there has not been a mechanical law which has not been verified by experience. There is no *a priori* law in Mechanics. In fact, there cannot be any. Mechanics must always rely on observation and experiment. Every science, limited to its own distinctive province, must necessarily be based on such assumptions as give its province the peculiar distinction that is all its own, or it is no science at all. Thus there is no rigid line of demarcation between the so-called deductive and inductive sciences. The two classes easily overlap. To seek to distinguish them is to attempt the impossible, and the attempt smacks of nineteenth century superstition. The utmost one can do in this line is to dub certain sciences as primarily deductive and certain others as primarily inductive, always bearing in mind, however, that the more a science advances onward, the more deductive does it become in both character and outlook for the very simple reason that it comes more and more to rely on the laws and principles it has discovered. Then again even such a purely deductive science as Pure Mathematics has often had to discard many of the assumptions with which it started on its career. Pure Mathematics is now no more what it used to be even a few decades ago.]

6. Clearly explain the utility of the Deductive Method.

7. What is Mill's deductive method? When is this method applicable? Give an example in which the deductive method is applied to prove induction.

8. Is the Deductive method applicable in all cases of induction? Fully discuss this question.

9. Name and explain the steps involved in the deductive method. Explain and illustrate the physical method, historical method and geometrical method.

10. Distinguish between the deduction employed by the method of residues and that employed by the deductive method.

11. What is the need of applying the deductive method in induction?

12. Show that combined induction and deduction expresses the full force of scientific method for resolving the greatest complications. Examine Mill's view of the deductive method.

13. What is intermixture of effects? Give an example. Show how intermixture of effects prevents the employment of the experimental methods. Do all the experimental methods fail in such a case? Give reasons for your answer. By what other method can the difficulty arising from intermixture of effects be overcome? Describe and illustrate the method fully.

14. When is the deductive method employed in inductive investigation? Distinguish between the direct and inverse forms of this method.

15. Explain the so-called deductive method. Describe the physical method, and give an example to show its application.

CHAPTERS VIII-IX

PROBABILITY, LAW AND EXPLANATION

1. Explain the nature of probability. How does it attempt to provide a rational account of the world by eliminating chance? Illustrate your answer.

2. How is probability concerned with mathematical calculation?

3. What is chance? Is there any room for chance in the universe? Why do men attribute events to chance?

4. Is probability merely subjective? What is its relation to induction? Do we require the calculation of probability in our daily life?

5. Explain what is meant by saying that calculations of probability are true of the average and in the long run. Is there any science based upon the calculation of probability? Illustrate your answer.

6. Should inductive generalisations be regarded as only probable?

7. What is quantitative determination? Can measurement be exact? Does improbability mean absence of probability?

8. What is the logical basis of probability? What is its province?

9. State and explain with examples the main rules for the calculation of probability.

10. How do we calculate the probability of simple events, independent events, dependent events, either of two events which cannot concur, recurrent events, and the value of cumulative evidence?

11. What is a law? Is the world governed by laws?

12. What is meant by the statement that the world is orderly and systematic?

13. Classify laws. Explain axiom, primary law, secondary law.

14. Distinguish between derivative laws and empirical laws. Give an example of each. Can an empirical law become derivative?

15. Explain with illustrations the statement that empirical laws are either of succession or of co-existence.

16. Explain the Baconian method. Is it useful from the scientific point of view?

17. Distinguish between invariable empirical laws and approximate empirical laws, with illustrations. Are primary or secondary laws of greater use in our daily life?

18. What is explanation? Explain the relation between hypothesis, explanation, and induction.

19. Can analogy or induction by simple enumeration offer scientific explanation?

20. "Scientific explanation consists in discovering, deducing and assimilating the laws of phenomena." Explain.

21. What is demonstration? Is demonstration a form of explanation? Distinguish between scientific and popular explanation.

22. Name and explain with illustrations the forms of explanation.

23. What are the limits of explanation? Does explanation show that the world is a system?

24. What is meant by demonstration? What kinds of inference are of demonstrative character and what kinds are merely probable? Explain the reason in each case, and give examples.

25. What are laws of Nature? Define and exemplify ultimate, secondary, derivative, and empirical laws, showing their relation to one another. To which class will those laws belong which are founded on the method of agreement? Give your reasons with examples.

26. What is a law? Distinguish between a law of the State, a law of Nature, and a logical law, illustrating your meaning with examples. Science must assume that Nature is subject to law: explain why it must do so.

27. Is there such a thing as chance? Discuss the relation between chance and causal connection, and indicate what is meant by calculation of probabilities.

28. Explain the following:—(*a*) The event A is probable. (*b*) The probability of the event A is 1/6. (*c*) The events A and B occur together by chance.

29. What is a law of Nature? How does it differ from an empirical law? Explain the use for science of the discovery of empirical laws.

30. What is meant by scientific explanation? In what sense is analogy described as incomplete explanation?

CHAPTER X

1. Define division and classification and explain the relation that exists between them.

2. How are division and classification related to definition?

3. Explain with examples the rules of division and classification. Name and exemplify the fallacies which arise from their violation.

4. Explain the relation between names and classification. Distinguish between artificial and scientific classification, and give examples. Is artificial classification in any way useful?

5. What is meant by natural classification and how is it related to scientific and artificial classification? Is natural classification possible?

6. What is the meaning of classification by type? Is

such classification scientific and satisfactory from the logical point of view?

7. What is meant by classification by series? Explain with examples.

8. What is meant by Nomenclature and Terminology? Explain their nature and utility with examples.

9. Distinguish between classification by type, by definition and by series. Which of them is scientific?

10. Is all classification artificial? What is a natural kind?

11. What is meant by a natural kind or class? Give an account of natural classification, explaining what is meant by essential or fundamental characters as the basis of classification. 'A class is nothing but the objects contained under it', examine this statement of Mill, showing whether it is correct or not.

12. Fully explain and illustrate the use of nomenclature and terminology. Exhibit the relation of nomenclature to definition and classification.

13. Scientific classification is classification by definition and not by type. Explain.

14. Is natural classification in any sense artificial?

CHAPTERS XI-XIII

1. Define method and clearly explain its nature.

2. What is analytic method and what is synthetic method?

3. What is methodology?

4. State and explain Descartes' general rules of method.

5. What is the essential nature of method?

6. Define analytic method and clearly explain its nature. What are the conditions of successful analysis?

7. Define synthetic method. Clearly explain its nature. Compare analytic and synthetic method.

8. State and explain the rules of synthetic method.

9. Distinguish between obscure and clear knowledge,

distinct and confused knowledge, adequate and inadequate knowledge, and symbolic and intuitive knowledge. When is knowledge perfect?

10. What is exposition and what is discovery? Explain the method employed in exposition and that employed in discovery.

11. Name the sciences which require synthetic method and those that require analytic method, and state why they do so. Is there any science which requires both these methods? What is the nature of the reasoning involved in synthetic method and of that involved in analytic method?

12. Explain the nature of the method or methods employed in the following sciences:—Physics, Chemistry, Mathematics, Botany, Biology, Economics, Politics, Sociology and History.

13. Explain the nature of historical, comparative and evolutionary method.

CHAPTERS XIV-XV

1. Give a short survey of the development of inductive logic.

2. Why is Bacon regarded as the father of induction? Give a short account of Bacon's view of scientific method.

3. Compare Whewell's standpoint with regard to induction with that of Mill.

4. Explain with illustrations the fallacies of mal-observation and non-observation. What is the difference between them?

5. What are the causes of non-observation and mal-observation?

6. Explain with illustrations the fallacy of false generalisation. Why is this fallacy committed?

7. Explain with illustrations the fallacy of post hoc ergo propter hoc.

8. Explain with illustrations the fallacy of false analogy.

9. Explain with illustrations the fallacy incident to explanation. What is illusory explanation?

10. Explain with illustrations the fallacy of petitio principii. Name and explain its sub-forms.

11. What is meant by begging the question? Explain with illustrations the fallacies of hysteron proteron and circulus in demonstrando.

12. Clearly explain the nature of ignoratio elenchi. Name and explain its sub-forms.

13. Explain with illustrations the following fallacies:—Argumentum ad misericordiam, Argumentum ad hominem, Argumentum ad baculum, Argumentum ad populum, Argumentum ad ignoratiam, Argumentum ad verecundiam, Non-causa pro-causa, and Non-sequitur.

14. (a) An eclipse of the sun will occur when the moon intervenes between the earth and the sun; an eclipse of the sun will occur when some great calamity is impending over mankind. Examine the logical grounds and comparative validity of the above two propositions. (b) Napoleon's Russian expedition was the cause of his downfall. Explain the fallacy here.

15. When beggars die there are no comets seen; the heavens themselves blaze forth the death of princes. Characterise logically the grounds of this belief.

16. (a) The Terror ceased immediately on the death of Robespierre; therefore Robespierre was the cause of the Terror. Examine this. (b) Yesterday the smoke of the chimneys tended to sink downwards and it rained in the afternoon: can any connection be inferred from this?

17. Test the following:—

(a) The people of England are wealthy because they are industrious.

(b) If justice consists in keeping property, the just man must be a kind of thief; for the same kind of skill which enables a man to defend property will also enable him to steal it.

(c) As soon as I sat down to study this morning, the man in the adjoining room began to play on the harmonium. He must therefore be a very malicious person.

(d) This patent medicine must be very efficacious, for all the testimonials speak of the marvellous cures effected by it.

by the law of the survival of the fittest, those that are still alive must be fitter and better than those that are gone.

(d) I do not consult physicians, for those that do so also die.

26. Test the following:—

(a) Linnets, when shut up and educated with singing larks, the sky-lark, wood-lark, or tit-lark, will adhere entirely to the songs of these larks instead of the natural song of the linnets. Hence we may infer that birds learn to sing by imitation, and their songs are no more innate than language is in man.

(b) Vesalius, the founder of modern anatomy, found that the human thigh-bone was straight, and not curved, as Galen, the great authority on the subject for over a thousand years, had asserted. Sylvius replied that Galen must be right; that the bone was curved in its natural condition, but that the narrow trousers worn at the time had made it artificially straight.

(c) States that have grown outrageously luxurious have declined in power. Hence we conclude that luxury was the cause of their downfall.

27. Examine the following:—

(a) All bats are birds, for they have wings.

(b) Wine cannot be injurious to health, for if it were so, the doctor would not have prescribed it.

(c) The anatomical resemblance between men and apes is marvellous, and from such resemblance we can safely conclude that men are descended from apes.

(d) My friend must be a genius, for he has many eccentricities, as all geniuses have.

(e) The professor must be a very learned man, for his words are so big and hard that very few understand them.

28. Give an example of each of the following fallacies:—Post hoc ergo propter hoc, Mistaking an inference for an observed fact, Mistaking a condition for a cause.

29. Test the validity of the following inductive arguments, giving reasons and naming the experimental method by which each is established:—

(a) A conjurer produces wonderful results by different tricks on different occasions, taking care to wave his wand in

each case. Therefore the waving of his wand is the cause of the wonderful results.

(b) As soon as I came to this place my disease was cured. Therefore the climate of this place effected the cure of my disease.

30. Test the following arguments:—

(a) Scarlet poppies, scarlet verbenas, the scarlet hawthorn and honeysuckle are all odourless; therefore we may conclude that all scarlet flowers are destitute of odour.

(b) The planet Mars resembles the earth in possessing an atmosphere, water, and moderate temperature, and we may therefore suppose it to be inhabited.

31. Test the validity of the following arguments, naming the fallacy (if any) and stating reasons in each case:—

(a) Women as a class have not hitherto been equal to men, therefore they are necessarily inferior to men.

(b) Education is clearly the source of all discontent, since the educated, not getting suitable employment, are dissatisfied with their lot.

32. Test the validity of the following arguments:—

(a) The eating of mangoes is the cause of boils.

(b) All religions lead to God, for do not all roads lead to Rome, and all rivers fall into the sea?

(c) The mind must be a function of the brain, since any serious injury to the brain is always followed by loss of consciousness.

(d) The University is the temple of learning, and therefore politics has no place in it.

(Also consult exercises on inductive methods)

THE END